RSS-Based Localization in Wireless Sensor Networks Using Convex Relaxation: Noncooperative and Cooperative Schemes

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***Abstract*—In this paper, we propose new approaches based on convex optimization to address the received signal strength (RSS)-based noncooperative and cooperative localization problems in wireless sensor networks (WSNs). By using an array of passive anchor nodes, we collect the noisy RSS measurements from radiating source nodes in WSNs, which we use to estimate the source positions. We derive the maximum likelihood (ML) estimator, since the ML-based solutions have particular importance due to their asymptotically optimal performance. However, the ML estimator requires the minimization of a nonconvex objective function that may have multiple local optima, thus making the search for the globally optimal solution hard. To overcome this difficulty, we derive a new nonconvex estimator, which tightly approximates the ML estimator for small noise. Then, the new estimator is relaxed by applying efficient convex relaxations that are based on second-order cone programming and semidefinite programming in the case of noncooperative and cooperative localization, respectively, for both cases of known and unknown source transmit power. We also show that our approaches work well in the case when the source transmit power and the path loss exponent are simultaneously unknown at the anchor nodes. Moreover, we show that the generalization of the new approaches for the localization problem in indoor environments is straightforward. Simulation results show that the proposed approaches significantly improve the localization accuracy, reducing the estimation error between 15% and 20% on average, compared with the existing approaches.**

***Index Terms*—Centralized localization, cooperative localization, noncooperative localization, received signal strength (RSS), second-order cone programming (SOCP) problem, semidefinite programming (SDP) problem, wireless localization, wireless sensor network (WSN).**

Manuscript received December 17, 2013; revised April 23, 2014 and June 19, 2014; accepted June 26, 2014. Date of publication July 1, 2014; date of current version May 12, 2015. This work was supported in part by the Fundação para a Ciência e a Tecnologia under Projects PEst-OE/EEI/UI0066/2014, EXPL/EEI-TEL/0969/2013-MANY2COMWIN and EXPL/EEI-TEL/1582/2013-GLANC, PEst-OE/EEI/LA0008/2013 (IT pluriannual founding and HETNET), PEst-OE/EEI/UI0066/2011 (UNINOVA pluriannual founding), EnAcoMIMOCo EXPL/EEI-TEL/2408/2013, and ADIN PTDC/EEI-TEL/2990/2012, and Grant SFRH/BD/91126/2012 and the Ciência 2008 Post-Doctoral Research grant. This paper was presented in part at the IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2013) Darmstadt, Germany, June 16–19, 2013. The review of this paper was coordinated by Prof. G. Mao.

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Digital Object Identifier 10.1109/TVT.2014.2334397

I. Introduction

Wireless sensor networks (WSNs) comprise a number of sensor nodes, which can, in general, be classified as anchor and source (target) nodes [1]. The locations of the anchor nodes are known, whereas the locations of the source nodes are yet to be determined. WSNs have application in various areas such as target tracking, intrusion detection, energy-efficient routing, monitoring, underground, deep water, outer space explorations, etc. [2]. In WSNs, sensor nodes are deployed over a monitored region to acquire some physical data about the environment, such as temperature, pressure, humidity, wind speed, etc. The collected information, together with the object's position information, enables us to develop intelligent systems. Such systems offer improved safety and efficiency in everyday life, since each individual device in the network can respond faster and better to the changes in the environment (e.g., location-aware vehicles and asset management in warehouses) [3]. The idea of wireless positioning was initially conceived for cellular networks, since it invokes many innovative applications and services for its users. Nowadays, rapid increase in heterogeneous smart devices (mobile phones and tablets), which offer self-sustained applications and seamless interfaces to various wireless networks, is pushing the role of the location information to become a crucial component for mobile context-aware applications [1].

Location information in WSNs is usually obtained by range-based or range-free measurements. In this paper, we focus only on the former, since they provide higher estimation accuracy in general. In the range-based localization process, the main concern is to accomplish good estimation accuracy from inaccurate position-bearing measurements collected inside the network. To obtain the information of interest, it is necessary to enable node communication, which can be noncooperative or cooperative. The former approach allows source nodes to communicate only with the anchor nodes, whereas the latter approach allows source nodes to communicate with all nodes inside their communication range, whether they are anchor or source nodes [1]. Algorithmically, both approaches can be executed in a distributed (self-positioning) or centralized (network-centric positioning) mode. Although the distributed approach has, in general, low complexity and high scalability, it is sensitive to error propagation and may require long convergence time. Therefore, in this paper, we concentrate only on the centralized approach. To establish an accurate estimate of the source position, the processor must have prior knowledge about the anchor nodes' positions. In cellular networks, the base stations can be seen as anchor nodes, and the mobile stations can be seen as source nodes.

Depending on the available hardware, current distance-based localization techniques exploit different measurements of the radio signal transmitted between nodes [received signal strength (RSS), time-difference-of-arrival (TDOA), time-of-arrival (TOA), roundtrip-time (RTT), or angle-of-arrival (AOA) measurements]. The tradeoff between the localization accuracy and the implementation complexity of each technique is a very important factor when deciding which method to employ. For example, localization based on TOA or TDOA [including the Global Positioning System (GPS)] gives high estimation accuracy, but it requires a very complex process of timing and synchronization, thus making the localization cost expensive [4]. Although less accurate than the localization using TOA, TDOA, or AOA information, localization based on the RSS measurements requires no specialized hardware, less processing, and communication (and, consequently, lower energy), thus making it an attractive low-cost solution for the localization problem [2], [5]. Another attractive low-cost approach might be exploiting RTT measurements, which are easily obtained in wireless local area network (WLAN) systems by using a simple device such as a printed circuit board [6]. Although RTT systems circumvent the problem of clock synchronization between nodes, the major drawback of this approach is the need for double signal transmission to perform a single measurement [7]. Recently, hybrid methods that fuse two measurements of the radio signal (e.g., RSS-RTT) attracted considerable attention in the research society [6], [8], [9]. These methods try to improve the estimation accuracy in node positions by exploiting the benefits of the combined measurements, together with minimizing their drawbacks. In this paper, we focus on providing a good localization accuracy by using RSS measurements exclusively.

A. Related Work

Source localization based on the RSS measurements has recently attracted much attention in the wireless communications community [10]–[18]. The most popular estimator used in practice is the maximum likelihood (ML) estimator, since it is asymptotically efficient (for large enough data records) [19]. However, solving the ML estimator of the RSS-based localization problem is a very difficult task, because it is highly nonlinear and noncovex [5]; hence, it may have multiple local optima. In this case, search for the globally optimal solution is very hard via iterative algorithms, since they may converge to a local minimum or a saddle point resulting in a large estimation error. To overcome this difficulty and possibly provide a good initial point (close to the global minimum) for the iterative algorithms, approaches such as grid search methods, linear estimators, and convex relaxation techniques have been introduced to address the ML problem [10]–[18]. The grid search methods are time consuming and require a huge amount of memory when the number of unknown parameters is too large. Linear estimators are very efficient in the sense of time consumption, but they are derived based on many approximations that may affect their performance, particularly in the case when the noise is large [14]. In convex relaxation techniques such as those in [12]–[18], the difficulties in the ML problem are overcome by transforming the original nonconvex and nonlinear problem into a convex problem. The advantage of this approach is that convergence to the globally optimal solution is guaranteed. However, due to the use of relaxation techniques, the solution of a convex problem does not necessarily correspond to the solution of the original ML problem [20].

In [10], different weighting schemes for the multidimensional scaling (MDS) formulation were presented and compared. It was shown that the solution of the MDS can be used as the initial value for iterative algorithms, which then converge faster and attain higher accuracy when compared with random initial values. Convex semidefinite programming (SDP) estimators were proposed in [12] to address the nonconvexity of the ML estimator, for both noncooperative and cooperative localization problems with known source transmit power, i.e., P_T. Ouyang *et al.* in [12] reformulated the localization problem by eliminating the logarithms in the ML formulation and approaching the localization problem as a minimax optimization problem, which is then relaxed as SDP. Although the approach described in [12] provides good estimation results, particularly for the case of cooperative localization, it has high computational complexity, which might restrict its application in large-scale WSNs. In [13], the RSS-based localization problem for known P_T was formulated as the weighted least squares (WLS) problem, based on the unscented transformation (UT). It was shown that for the cooperative localization, the WLS formulation can be relaxed as a mixed semidefinite and second-order cone programming problem (SD/SOCP), whereas for the noncooperative localization, the WLS problem can be solved by the bisection method. In [16], Wang *et al.* addressed the noncooperative RSS localization problem for the case of unknown P_T and the path loss exponent (PLE). For the case of unknown P_T, based on the UT, a WLS formulation of the problem is derived, which was solved by the bisection method. When both P_T and PLE are not known, an alternating estimation procedure is introduced. However, both [13] and [16] have the assumption of perfect knowledge of the noise standard deviation (STD). This might not be the case in practice, particularly in low-cost systems such as RSS where calibration is avoided due to maintaining low system costs [2], [5]. In [18], Vaghefi *et al.* addressed the RSS cooperative localization problem for unknown P_T. The case where the source nodes have different P_T (e.g., due to different antenna gains) was considered in [18]. The authors solved the localization problem by applying an SDP relaxation technique and converting the original ML problem into a convex problem. Furthermore, in [18], the authors examined the effect of imperfect knowledge of the PLE on the performance of the SDP algorithm and used an iterative procedure to solve the problem when P_T and PLE are simultaneously unknown.

B. Contributions

In this paper, the RSS-based source localization problem for both noncooperative and cooperative scenarios is considered. Instead of solving the ML problem, which is highly nonconvex and computationally exhausting to solve globally, we propose a suboptimal approach that provides an efficient solution. Hence, we introduce a new nonconvex least squares (LS) estimator that tightly approximates the ML estimator for small noise. This estimator represents a smoother and simpler localization problem in comparison to the ML problem. Applying appropriate convex relaxations to the derived nonconvex estimator, novel SOCP and novel mixed SDP/SOCP estimators are proposed for noncooperative and cooperative localization cases, respectively.

The proposed approach offers an advantage over the existing approaches as it allows straightforward adaptation to different scenarios of the RSS localization problem, thereby significantly reducing the estimation error. In both noncooperative and cooperative scenarios, we first consider the simplest case of the localization problem where P_T is known at the anchor nodes. Next, we consider a more realistic scenario in which we assume that P_T is an unknown parameter, and we generalize our approaches for this setting. Finally, we investigate the most challenging scenario of the localization problem when P_T and PLE are simultaneously unknown at the anchor nodes. In this case, for the noncooperative localization, we apply an iterative procedure based on the proposed SOCP method to estimate all unknown parameters. We also provide details about the computational complexity of the considered algorithms.

In contrast to [12] and [13] where the authors considered the localization problem for the case when P_T is known, here, we address a more challenging scenario when both P_T and PLE are not known. In [13] and [16], Wang and Yang and Wang *et al.*, respectively, assumed that accurate knowledge of the noise STD is available, which might not be a valid assumption in some practical scenarios. Hence, we consider a more realistic scenario in which the noise STD is not available. In contrast to [18] where an SDP estimator is derived for the case of unknown P_T, we derive our estimators by using SOCP relaxation for the noncooperative case and mixed SDP/SOCP relaxation for the cooperative case.

The remainder of this paper is structured as follows. In Section II, the RSS measurement model for locating a single source node is introduced, the source localization problem is formulated for the case of known source transmit power, and the development of the proposed SOCP estimator is presented. We then extend this approach for the case where P_T, and P_T and PLE are simultaneously unknown. Section III introduces the RSS measurement model for the cooperative localization where multiple source nodes are simultaneously located. We give the formulation of the cooperative localization problem and provide details on the development of the proposed SDP estimators for both cases of known and unknown source transmit power. The complexity analysis is summarized in Section IV. In Section V, we provide both the complexity and simulation results to compare the performance of the newly proposed estimators with the existing estimators. Finally, in Section VI, we summarize the main conclusions.

II. Noncooperative Localization Via Second-Order Cone Programming Relaxation

Let us consider a WSN with N anchors and one source, where the locations of the anchors are respectively denoted by $\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N$, and the location of the unknown source is denoted by $\boldsymbol{x}$. Without loss of generality, this paper focuses on the 2-D scenario, i.e., $\boldsymbol{x}, \boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N \in \mathbb{R}^2$ (the extension for a 3-D scenario is straightforward). For the sake of simplicity, we assume that all anchors are equipped with omnidirectional antennas and connected to the source. Furthermore, it is assumed that the anchor positions are known. Under the lognormal shadowing and log-distance path loss model, the path loss between the ith anchor and the unknown source, i.e., L_i, can be modeled according to the following radio propagation path loss model (in decibels) [22]–[25], i.e.,

$$L_i = L_0 + 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0} + v_i, \quad i = 1, \ldots, N \tag{1}$$

where L_0 denotes the path loss value at a short reference distance d_0 ($\|\boldsymbol{x} - \boldsymbol{s}_i\| \geq d_0$), γ is the PLE, and v_i is the lognormal shadowing term modeled as a zero-mean Gaussian random variable with variance σ_i^2, i.e., $v_i \sim \mathcal{N}(0, \sigma_i^2)$. The model has been validated by a variety of measurement results [23]–[28].

From the relationship L_i (dB) $= 10\log_{10}(P_T/P_i)$, where P_i is the RSS measured by the ith anchor, and P_T is the transmission power of the unknown source, it is easy to see that the localization problem can be formulated by the path loss instead of the RSS. Hence, as in [12] and [13], the path-loss-based approach is adopted in this paper. Based on the measurements in (1), the ML estimator is found by solving the nonlinear and nonconvex LS problem, i.e.,

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left[(L_i - L_0) - 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0} \right]^2. \tag{2}$$

To solve (2), recursive methods such as Newton's method, combined with the gradient descent method, are often used [19]. However, the objective function may have many local optima, and local search methods may easily get trapped in a local optimum. Hence, in this paper, we employ convex relaxation to address the nonconvexity of the localization problem.

The remainder of this section is organized as follows. Section II-A deals with the case where P_T is known, whereas Section II-B deals with the case where P_T is considered to be an unknown parameter that needs to be estimated. Finally, Section II-C addresses the more general problem when P_T and PLE are simultaneously unknown.

A. Noncooperative Scenario With Known P_T

The source might be designed to measure and report its own calibration data to the anchors, in which case it is reasonable to assume that the source transmission power is known [5]. This corresponds to the case when the reference path loss L_0, which depends on P_T [22], is known.

For the sake of simplicity, in the rest of this paper, we assume that $\sigma_i^2 = \sigma^2$, for $i = 1, \ldots, N$. When the noise is sufficiently small, from (1), we get

$$\alpha_i \|\boldsymbol{x} - \boldsymbol{s}_i\| \approx d_0 \tag{3}$$

where $\alpha_i = 10^{(L_0 - L_i)/10\gamma}$. One way for estimating the source location $\boldsymbol{x}$ is via the minimization of the LS criterion. Thus, according to (3), the LS estimation problem can be formulated as[1]

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} \left(\alpha_i \|\boldsymbol{x} - \boldsymbol{s}_i\| - d_0\right)^2. \tag{4}$$

Although the problem in (4) is nonconvex, when $\alpha_i = d_0$, for $i = 1, \ldots, N$, it can be accurately solved by the SCLP method presented in [30].[2] In the following text, we will present a novel approach to solve the problem defined in (4).

Defining auxiliary variables $\boldsymbol{z} = [z_1, \ldots, z_N]^T$, where $z_i = \alpha_i g_i - d_0$, and $g_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|$, from (4), we get

$$\underset{\boldsymbol{x},\boldsymbol{g},\boldsymbol{z}}{\text{minimize}} \ \|\boldsymbol{z}\|^2$$

subject to

$$g_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|, \ z_i = \alpha_i g_i - d_0, \ i = 1, \ldots, N. \tag{5}$$

Introducing an epigraph variable t and relaxing the nonconvex constraint $g_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|$ as $g_i \geq \|\boldsymbol{x} - \boldsymbol{s}_i\|$ yield the following SOCP problem:

$$\underset{\boldsymbol{x},\boldsymbol{g},\boldsymbol{z},t}{\text{minimize}} \ t$$

subject to

$$\begin{aligned} \|[2\boldsymbol{z}; t-1]\| &\leq t+1, \ \|\boldsymbol{x} - \boldsymbol{s}_i\| \leq g_i, \\ z_i &= \alpha_i g_i - d_0, \ i = 1, \ldots, N. \end{aligned} \tag{6}$$

Problem (6) can be efficiently solved by [21], and we will refer to it as "SOCP1" in the following text.[3]

B. Noncooperative Scenario With Unknown P_T

The assumption that the anchors know the actual source transmission power may be too strong in practice since it would require additional hardware in both source and anchors [5]. Here, a more realistic and challenging scenario where the anchor nodes are not aware of the source transmission power is considered; thus, L_0 is assumed to be unknown and has to be estimated. The joint ML estimation of $\boldsymbol{x}$ and L_0 can be formulated as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}=[\boldsymbol{x};L_0]} \sum_{i=1}^{N} \frac{1}{\sigma^2} \left[(L_i - \boldsymbol{l}^T\boldsymbol{\theta}) - 10\gamma \log_{10} \frac{\|\boldsymbol{A}^T\boldsymbol{\theta} - \boldsymbol{s}_i\|}{d_0} \right]^2 \tag{7}$$

where $\boldsymbol{l} = [\boldsymbol{0}_{2\times 1};\ 1]$, and $\boldsymbol{A} = [\boldsymbol{I}_2;\ \boldsymbol{0}_{1\times 2}]$.

In (3), we assumed that P_T, i.e., L_0 is known. Assuming that L_0 is unknown, we can rewrite (3) as

$$\psi_i \, \|\boldsymbol{x} - \boldsymbol{s}_i\| \approx \eta d_0 \tag{8}$$

where $\psi_i = 10^{-L_i/10\gamma}$, and $\eta = 10^{-L_0/10\gamma}$. By following a procedure similar to that in Section II-A, we obtain the SOCP problem, i.e.,

$$\underset{\boldsymbol{x},\boldsymbol{g},\boldsymbol{z},\eta,t}{\text{minimize}} \ t$$

subject to

$$\begin{aligned} \|[2\boldsymbol{z}; t-1]\| &\leq t+1, \ \|\boldsymbol{x} - \boldsymbol{s}_i\| \leq g_i \\ z_i &= \psi_i g_i - \eta \, d_0, \ i = 1, \ldots, N. \end{aligned} \tag{9}$$

Although the approach in (9) efficiently solves (7), we can further improve its performance. To do so, we will exploit the estimate of L_0, i.e., $\widehat{L}_0$, which we get by solving (9), and solve another SOCP problem. This SOCP approach will be described in the following text.

Introducing auxiliary variables $r_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|$ and $\gamma_i = r_i^2$, expanding (4), and dropping the term d_0^2, which has no effect on the minimization, yield

$$\begin{aligned} &\underset{\boldsymbol{x},\boldsymbol{\gamma},\boldsymbol{r}}{\text{minimize}} \quad \sum_{i=1}^{N} \left(\widehat{\alpha}_i^2 \gamma_i - 2d_0 \widehat{\alpha}_i r_i\right) \\ &\text{subject to } \gamma_i = r_i^2, \quad r_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|, \ i = 1, \ldots, N \end{aligned} \tag{10}$$

where $\widehat{\alpha}_i = 10^{(L_i - \widehat{L}_0)/10\gamma}$. One can relax (10) to a convex optimization problem as follows. The nonconvex constraint $\gamma_i = r_i^2$ will be replaced by the second-order cone constraint (SOCC) $r_i^2 \leq \gamma_i$. In fact, the inequality constraint $r_i^2 \leq \gamma_i$ will be satisfied as an equality since γ_i and r_i will decrease and increase in the minimization, respectively. Furthermore, define an auxiliary variable $y = \|\boldsymbol{x}\|^2$. The constraint $y = \|\boldsymbol{x}\|^2$ is relaxed to a convex constraint $y \geq \|\boldsymbol{x}\|^2$, which is evidently an SOCC. With the use of all developed constraints, problem (10) is approximated as a convex, SOCP, optimization problem, i.e.,

$$\underset{\boldsymbol{x},\boldsymbol{\gamma},\boldsymbol{r},y}{\text{minimize}} \ \sum_{i=1}^{N} \left(\widehat{\alpha}_i^2 \gamma_i - 2d_0 \widehat{\alpha}_i r_i\right)$$

subject to

$$\begin{aligned} \|[2\boldsymbol{x}; y-1]\| &\leq y+1, \ \|[2r_i; \gamma_i - 1]\| \leq \gamma_i + 1 \\ \gamma_i &= y - 2\boldsymbol{s}_i^T\boldsymbol{x} + \|\boldsymbol{s}_i\|^2, \ i = 1, \ldots, N. \end{aligned} \tag{11}$$

[1] A justification for dropping the shadowing term in the propagation path loss model is provided in the following text. We can rewrite (1) as $(L_i - L_0)/10\gamma = \log_{10}(\|\boldsymbol{x} - \boldsymbol{s}_i\|/d_0) + (v_i/10\gamma)$, which corresponds to $\alpha_i\|\boldsymbol{x} - \boldsymbol{s}_i\| = d_0 10^{v_i/10\gamma}$. For sufficiently small noise, the first-order Taylor series expansion to the right-hand side of the previous expression is given by $\alpha_i\|\boldsymbol{x} - \boldsymbol{s}_i\| = d_0(1 + (\ln 10/10\gamma)v_i)$, i.e., $\alpha_i\|\boldsymbol{x} - \boldsymbol{s}_i\| = d_0 + \epsilon_i$, where $\epsilon_i = d_0(\ln 10/10\gamma)v_i$ is the zero-mean Gaussian random variable with the variance $d_0^2((\ln 10)^2/100\gamma^2)\sigma^2$. Clearly, the corresponding LS estimator is given by (4). The same has been done in [18].

[2] It is possible to generalize the SCLP method to the weighted case, i.e., to the case when $\alpha_i \neq d_0$ for some i. However, the algorithm in [30] yields a meaningless solution. This is due to the fact that the *almost convexity* property of the resulting constraints is not preserved.

[3] It is worth noting that the "SOCP1" approach can be modified to solve the localization problem in a distributed fashion.

In summary, the proposed procedure for solving (7) is given as follows.

Step 1) Solve (9) to obtain the initial estimate of $\boldsymbol{x}$, $\hat{\boldsymbol{x}}'$.

Step 2) Use $\hat{\boldsymbol{x}}'$ to compute the ML estimate of L_0, $\widehat{L}_0'$, from (7) as

$$\widehat{L}_0' = \frac{\sum_{i=1}^{N}\left(L_i - 10\gamma\log_{10}\frac{\|\hat{\boldsymbol{x}}'-\boldsymbol{s}_i\|}{d_0}\right)}{N}. \tag{12}$$

Step 3) Use $\widehat{L}_0'$ to solve the SOCP in (11) and obtain the new source position estimate $\boldsymbol{x}$, $\hat{\boldsymbol{x}}''$. Compute the ML estimate of L_0, $\widehat{L}_0''$, from (12), by using $\hat{\boldsymbol{x}}''$.

The main reason for applying this simple procedure is that we observed in our simulations that after solving (9), we obtain an excellent ML estimation of L_0, i.e., $\widehat{L}_0'$, which is very close to the true value of L_0. This motivated us to take advantage of this estimated value and solve another SOCP problem (11), as if P_T, i.e., L_0 is known. In Section V-A2, we will see remarkable improvements in the estimation accuracy of both $\boldsymbol{x}$ and L_0 by employing the given procedure. We denote this three-step procedure as "SOCP2."

C. Noncooperative Scenario With Unknown P_T and γ

Signal attenuation may be caused by many effects, such as multipath fading, diffraction, reflection, environment, and weather condition characteristics. Thus, it is reasonable to assume that PLE, i.e., γ, is not known at the anchor nodes. Here, we investigate the case where P_T and γ are simultaneously unknown at the anchor nodes. The joint ML estimation of $\boldsymbol{x}$, L_0, and γ is written as

$$\hat{\boldsymbol{\theta}}=\underset{\boldsymbol{\theta}=[\boldsymbol{x};L_0;\gamma]}{\arg\min}\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\left[(L_i-\boldsymbol{h}^T\boldsymbol{\theta})-10\boldsymbol{g}^T\boldsymbol{\theta}\log_{10}\frac{\|\boldsymbol{C}^T\boldsymbol{\theta}-\boldsymbol{s}_i\|}{d_0}\right]^2 \tag{13}$$

where $\boldsymbol{h}=[\boldsymbol{0}_{2\times1};\ 1;\ 0]$, $\boldsymbol{g}=[\boldsymbol{0}_{3\times1};\ 1]$, and $\boldsymbol{C}=[\boldsymbol{I}_2;\ \boldsymbol{0}_{2\times2}]$. Problem (13) is nonconvex and has no closed-form solution. To tackle (13), we employ a standard alternating procedure explained as follows (see also [16] and the references therein).

Step 1) Instead of blind estimation, use empirical values, e.g., [22], and set the initial estimate of γ, $\hat{\gamma}^0\in[\gamma_{\min},\gamma_{\max}]$, and solve (9) to find the initial estimate of $\boldsymbol{x}$, $\hat{\boldsymbol{x}}^0$. Use $\hat{\gamma}^0$ and $\hat{\boldsymbol{x}}^0$ to calculate the ML estimate of L_0, $\widehat{L}_0^0$. Compute the value of the objective function, i.e., f_0, by plugging $\{\hat{\boldsymbol{x}}^0,\widehat{L}_0^0,\hat{\gamma}^0\}$ into (13). Set $k=1$.

Step 2) Use $\hat{\boldsymbol{x}}^{k-1}$ and $\widehat{L}_0^{k-1}$ to find the ML estimate of γ, $\widehat{\gamma}^k$, as

$$\widehat{\gamma}^k = \frac{\sum_{i=1}^{N}10\log_{10}\frac{\|\hat{\boldsymbol{x}}^{k-1}-\boldsymbol{s}_i\|}{d_0}\left(L_i-\widehat{L}_0^{k-1}\right)}{\sum_{i=1}^{N}\left(10\log_{10}\frac{\|\hat{\boldsymbol{x}}^{k-1}-\boldsymbol{s}_i\|}{d_0}\right)^2}.$$

If $\widehat{\gamma}^k\in[\gamma_{\min},\gamma_{\max}]$ go to step 3); else stop.

Step 3) Use $\widehat{\gamma}^k$ and $\widehat{L}_0^{k-1}$ to solve (11), obtain the estimate of $\boldsymbol{x}$, $\hat{\boldsymbol{x}}^k$, and update the ML estimate of L_0, $\widehat{L}_0^k$. Plug $\{\hat{\boldsymbol{x}}^k,\widehat{L}_0^k,\hat{\gamma}^k\}$ into (13) and compute the value of the objective function, i.e., f_k. If $|f_k-f_{k-1}|/f_{k-1}<\epsilon$ (ϵ is a small positive number) or $k>K_{\max}$ ($K_{\max}$ is the maximum number of iteration) stop; otherwise let $k=k+1$ and go to step 2).

We refer to the given described iterative procedure as "SOCP3" in this paper.

III. Cooperative Localization Via Semidefinite Programming Relaxation

Consider now a WSN with N anchors and M source nodes, where, as before, the locations of the anchors $\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_N$ are known, and the locations of the source nodes are $\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_M$ (where $\boldsymbol{x}_i,\boldsymbol{s}_j\in\mathbb{R}^2$, for $i=1,\ldots,M$ and $j=1,\ldots,N$). Due to the limited communication range, i.e., R, or other physical limitations, only some source nodes can directly connect to the anchor nodes, making the information gathered inside the network insufficient to perform a good estimation. To overcome this problem, node cooperation is required. Node cooperation allows direct communication between any two nodes in the WSN, which are within the communication range of each other. This means that the source nodes also perform RSS measurements, i.e., they play a role of the anchor nodes, which number is scarce, to acquire adequate amount of information. After the RSS measurements are collected, all locations of the source nodes are simultaneously estimated. This kind of localization is called cooperative localization [1].

For ease of expression, matrix $\boldsymbol{X}$ is built such that it contains the positions of all sources, i.e., $\boldsymbol{X}=[\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_M]$ ($\boldsymbol{X}\in\mathbb{R}^{2\times M}$). Furthermore, sets $\mathcal{A}=\{(i,j):\|\boldsymbol{x}_i-\boldsymbol{s}_j\|\le R,\quad i=1,\ldots,M, j=1,\ldots,N\}$, and $\mathcal{B}=\{(i,k):\|\boldsymbol{x}_i-\boldsymbol{x}_k\|\le R,\ i,k=1,\ldots,M,\ i\neq k\}$ denote the existence of the source/anchor and the source/source connections, respectively. For the sake of simplicity, we assume that all source nodes radiate with the same power P_T, i.e., L_0 and R are the same for all source nodes.

According to the radio propagation path loss model in [22], the RSS measurement for the cooperative localization can be formulated as

$$\begin{aligned} L_{ij}^{\mathcal{A}} &= L_0+10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{s}_j\|}{d_0}+v_{ij},\ (i,j)\in\mathcal{A}\\ L_{ik}^{\mathcal{B}} &= L_0+10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{x}_k\|}{d_0}+w_{ik},\ (i,k)\in\mathcal{B} \end{aligned} \tag{14}$$

where v_{ij} and w_{ik} are the lognormal shadowing terms modeled as zero-mean Gaussian random variables with variances $\sigma_{v_{ij}}^2$ and $\sigma_{w_{ik}}^2$, i.e., $v_{ij}\sim\mathcal{N}(0,\sigma_{v_{ij}}^2)$ and $w_{ik}\sim\mathcal{N}(0,\sigma_{w_{ik}}^2)$. We assume that the source/source path loss measurements are symmetric, i.e., $L_{ik}^{\mathcal{B}}=L_{ki}^{\mathcal{B}}$ for $i\neq k$.

As in (1), the path loss approach and the measurements in (14) lead to the ML estimator, corresponding to solving the nonlinear and nonconvex LS problem, i.e.,

$$\begin{aligned}\hat{\boldsymbol{X}}=\underset{\boldsymbol{X}}{\arg\min}&\sum_{(i,j):(i,j)\in\mathcal{A}}\frac{1}{\sigma_{v_{ij}}^2}\left[(L_{ij}^{\mathcal{A}}-L_0)-10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{s}_j\|}{d_0}\right]^2\\ &+\sum_{(i,k):(i,k)\in\mathcal{B}}\frac{1}{\sigma_{w_{ik}}^2}\left[(L_{ik}^{\mathcal{B}}-L_0)-10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{x}_k\|}{d_0}\right]^2.\end{aligned} \tag{15}$$

The problem defined in (15) is nonconvex and nonlinear and, as far as we know, has no closed-form solution. As before, from (15), we distinguish two different cases. In the first case, P_T (i.e., L_0) is assumed to be known, whereas in the second case, P_T is considered to be an unknown parameter that has to be estimated. In Sections III-A and III-B, a centralized localization algorithm based on SDP relaxation will be presented to address problem (15) for the case of known and unknown P_T, respectively.

A. Cooperative Scenario With Known P_T

For the sake of simplicity, we assume that $\sigma_{v_{ij}}^2 = \sigma_{w_{ik}}^2 = \sigma^2$ in the remainder of this paper. The following procedures similar to those in the noncooperative localization problem, a convex estimator for cooperative localization is derived by applying semidefinite relaxation to the nonconvex problem (15).

As in Section II-A, we can approximate (14) as

$$\alpha_{ij}^{\mathcal{A}\,2}\|\boldsymbol{x}_i - \boldsymbol{s}_j\|^2 \approx d_0^2,\ \alpha_{ik}^{\mathcal{B}\,2}\|\boldsymbol{x}_i - \boldsymbol{x}_k\|^2 \approx d_0^2 \tag{16}$$

where $\alpha_{ij}^{\mathcal{A}} = 10^{(L_0 - L_{ij}^{\mathcal{A}})/10\gamma}$, and $\alpha_{ik}^{\mathcal{B}} = 10^{(L_0 - L_{ik}^{\mathcal{B}})/10\gamma}$. From (16), the following LS minimization problem is derived:

$$\underset{\boldsymbol{X}}{\text{minimize}} \sum_{(i,j):(i,j)\in\mathcal{A}} \left(\alpha_{ij}^{\mathcal{A}\,2}\|\boldsymbol{x}_i - \boldsymbol{s}_j\|^2 - d_0^2\right)^2 + \sum_{(i,k):(i,k)\in\mathcal{A}} \left(\alpha_{ik}^{\mathcal{B}\,2}\|\boldsymbol{x}_i - \boldsymbol{x}_k\|^2 - d_0^2\right)^2. \tag{17}$$

Next, define vector $\boldsymbol{y} = \text{vec}(\boldsymbol{X})$, where $\text{vec}(\boldsymbol{X})$ denotes the column-wise vectorization of $\boldsymbol{X}$. Then, (17) can be written as

$$\underset{\boldsymbol{y}}{\text{minimize}} \sum_{(i,j):(i,j)\in\mathcal{A}} \left(\alpha_{ij}^{\mathcal{A}\,2}\left\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j\right\|^2 - d_0^2\right)^2 + \sum_{(i,k):(i,k)\in\mathcal{B}} \left(\alpha_{ik}^{\mathcal{B}\,2}\left\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y}\right\|^2 - d_0^2\right)^2 \tag{18}$$

where $\boldsymbol{E}_i = [\boldsymbol{e}_{2i-1}, \boldsymbol{e}_{2i}]$, and $\boldsymbol{e}_i$ represents the ith column of the identity matrix $\boldsymbol{I}_{2M}$. Introducing an epigraph variable t and auxiliary variables $\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T$ and $\boldsymbol{z} = [z_{ij}^{\mathcal{A}}, z_{ik}^{\mathcal{B}}]^T$, where $z_{ij}^{\mathcal{A}} = \alpha_{ij}^{\mathcal{A}\,2}\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j\|^2 - d_0^2$, for $(i,j)\in\mathcal{A}$, and $z_{ik}^{\mathcal{B}} = \alpha_{ik}^{\mathcal{B}\,2}\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y}\|^2 - d_0^2$, for $(i,k)\in\mathcal{B}$, together with the convex relaxation $\boldsymbol{Y} \succeq \boldsymbol{y}\boldsymbol{y}^T$, the following convex epigraph form is obtained from (18):

$$\underset{\boldsymbol{y},\boldsymbol{Y},\boldsymbol{z},t}{\text{minimize}}\ t$$

subject to

$$\begin{aligned}
&z_{ij}^{\mathcal{A}} = \alpha_{ij}^{\mathcal{A}\,2}\left(\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i\right) - 2s_j^T\boldsymbol{E}_i^T\boldsymbol{y} + \|s_j\|^2\right) - d_0^2\\
&\quad \text{for } (i,j)\in\mathcal{A}\\
&z_{ik}^{\mathcal{B}} = \alpha_{ik}^{\mathcal{B}\,2}\left(\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i\right) - 2\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_k\right) + \text{tr}\left(\boldsymbol{E}_k^T\boldsymbol{Y}\boldsymbol{E}_k\right)\right) - d_0^2\\
&\quad \text{for } (i,k)\in\mathcal{B}\\
&\|[2\boldsymbol{z}; t-1]\| \le t+1,\ [\boldsymbol{Y}\ \boldsymbol{y}; \boldsymbol{y}^T\ 1] \succeq \boldsymbol{0}_{2M+1}.
\end{aligned} \tag{19}$$

The given problem is SDP (more precisely, it is mixed SDP/SOCP), which can be readily solved by `CVX` [21]. If $\text{rank}(\boldsymbol{Y}) = 1$, then constraint $\boldsymbol{Y} \succeq \boldsymbol{y}\boldsymbol{y}^T$ is satisfied as an equality [20]. Note that we applied the Schur complement to rewrite $\boldsymbol{Y} \succeq \boldsymbol{y}\boldsymbol{y}^T$ into a semidefinite cone constraint form [20]. Note also that, in huge contrast to the existing SDP-based approaches [12], [13], [16], [18], which consider the unknown parameters as a matrix, here, we consider them as a vector. In the remainder of this paper, we will denote the given approach as "SDP1."

B. Cooperative Scenario With Unknown P_T

The joint ML estimation of $\boldsymbol{X}$ and L_0 is given by

$$\begin{aligned}
\hat{\boldsymbol{\theta}} = &\underset{\boldsymbol{\theta}=[\boldsymbol{x}_1^T,\ldots,\boldsymbol{x}_M^T,L_0]^T}{\arg\min} \sum_{(i,j):(i,j)\in\mathcal{A}} \frac{1}{\sigma^2}\\
&\times\left[\left(L_{ij}^{\mathcal{A}} - \boldsymbol{q}^T\boldsymbol{\theta}\right) - 10\gamma\log_{10}\frac{\left\|\boldsymbol{A}_i^T\boldsymbol{\theta} - \boldsymbol{s}_j\right\|}{d_0}\right]^2\\
&+ \sum_{(i,k):(i,k)\in\mathcal{B}} \frac{1}{\sigma^2}\left[\left(L_{ik}^{\mathcal{B}} - \boldsymbol{q}^T\boldsymbol{\theta}\right) - 10\gamma\log_{10}\frac{\left\|\boldsymbol{A}_i^T\boldsymbol{\theta} - \boldsymbol{A}_k^T\boldsymbol{\theta}\right\|}{d_0}\right]^2
\end{aligned} \tag{20}$$

where $\boldsymbol{q} = [\boldsymbol{0}_{2M\times 1};\ 1]$, $\boldsymbol{A}_i = [\boldsymbol{r}_{2i-1},\ \boldsymbol{r}_{2i}]$, and $\boldsymbol{r}_i$ represents the ith column of the identity matrix $\boldsymbol{I}_{2M+1}$.

To solve (20), we will use a similar idea as in the "SDP1" approach. We can rewrite (16) as

$$\beta_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i - \boldsymbol{s}_j\|^2 \approx \eta d_0^2,\ \beta_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i - \boldsymbol{x}_k\|^2 \approx \eta d_0^2 \tag{21}$$

where $\beta_{ij}^{\mathcal{A}} = 10^{-L_{ij}^{\mathcal{A}}/5\gamma}$, $\beta_{ik}^{\mathcal{B}} = 10^{-L_{ik}^{\mathcal{B}}/5\gamma}$, and $\eta = 10^{-L_0/5\gamma}$. Following the steps highlighted in Section III-A, the following convex problem is obtained:

$$\underset{\boldsymbol{y},\boldsymbol{Y},\boldsymbol{z},t,\eta}{\text{minimize}}\ t$$

subject to

$$\begin{aligned}
&z_{ij}^{\mathcal{A}} = \beta_{ij}^{\mathcal{A}}\left(\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i\right) - 2s_j^T\boldsymbol{E}_i^T\boldsymbol{y} + \|s_j\|^2\right) - \eta d_0^2\\
&\quad \text{for } (i,j)\in\mathcal{A}\\
&z_{ik}^{\mathcal{B}} = \beta_{ik}^{\mathcal{B}}\left(\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i\right) - 2\text{tr}\left(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_k\right) + \text{tr}\left(\boldsymbol{E}_k^T\boldsymbol{Y}\boldsymbol{E}_k\right)\right) - \eta d_0^2\\
&\quad \text{for } (i,k)\in\mathcal{B}\\
&\|[2\boldsymbol{z}; t-1]\| \le t+1,\ [\boldsymbol{Y}\ \boldsymbol{y}; \boldsymbol{y}^T\ 1] \succeq \boldsymbol{0}_{2M+1}.
\end{aligned} \tag{22}$$

Although the approach in (22) efficiently solves (20), we can further improve its performance. As in Section II-B, we propose a simple three-step procedure as follows.

Step 1) Solve (22) to obtain the initial estimate of $\boldsymbol{y}$, $\hat{\boldsymbol{y}}'$.

Step 2) Use $\hat{\boldsymbol{y}}'$ to compute the ML estimate of L_0, $\widehat{L}_0'$, from (20) as

$$\widehat{L}_0' = \frac{\sum_{(i,j):(i,j)\in\mathcal{A}} \left(L_{ij}^{\mathcal{A}} - 10\gamma \log_{10} \frac{\| \boldsymbol{E}_i^T \hat{\boldsymbol{y}}' - \boldsymbol{s}_j \|}{d_0} \right)}{|\mathcal{A}| + |\mathcal{B}|} + \frac{\sum_{(i,k):(i,k)\in\mathcal{B}} \left(L_{ik}^{\mathcal{B}} - 10\gamma \log_{10} \frac{\| \boldsymbol{E}_i^T \hat{\boldsymbol{y}}' - \boldsymbol{E}_k^T \hat{\boldsymbol{y}}' \|}{d_0} \right)}{|\mathcal{A}| + |\mathcal{B}|} \tag{23}$$

where $|\mathcal{A}|$ and $|\mathcal{B}|$ represent the cardinalities of sets $\mathcal{A}$ and $\mathcal{B}$.

Step 3) Use $\widehat{L}_0'$ to solve (19) and obtain the new estimate of $\boldsymbol{y}$, $\hat{\boldsymbol{y}}''$. Compute the ML estimate of L_0, $\widehat{L}_0''$, from (23), by using $\hat{\boldsymbol{y}}''$.

We will refer to the given three-step procedure as "SDP2."

IV. Complexity Analysis

The tradeoff between the estimation accuracy and the computational complexity is one of the most important features of any method since it defines its applicability potential. This is the reason why, apart from the performance, we are interested in comparing the complexity of the proposed and the existing approaches.

The formula for computing the worst case complexity of mixed SD/SOCP [32], given in the following equation, is used to analyze the complexities of the considered algorithms in this paper:

$$\mathcal{O}\left(L \left(m \sum_{i=1}^{N_{\mathrm{sd}}} n_i^{\mathrm{sd}^3} + m^2 \sum_{i=1}^{N_{\mathrm{sd}}} n_i^{\mathrm{sd}^2} + m^2 \sum_{i=1}^{N_{\mathrm{soc}}} n_i^{\mathrm{soc}} + \sum_{i=1}^{N_{\mathrm{soc}}} n_i^{\mathrm{soc}^2} + m^3 \right) \right) \tag{24}$$

where L is the iteration complexity of the algorithm; m is the number of equality constraints; n_i^{sd} and n_i^{soc} are the dimensions of the ith semidefinite cone (SDC) and the ith second-order cone (SOC), respectively; and N_i^{sd} and N_i^{soc} are the number of SDC and SOC constraints, respectively. Equation (24) corresponds to the formula for computing the complexity of an SDP for the case when we have no SOCCs (in which case, L is the dimension of the SDP cone, given as a result of accumulating all SDP cones), and *vice versa* (in which case, L is the squared root of the total number of SOCCs) [32].

We investigated the worst case asymptotic complexity of the algorithms, i.e., we present only the dominating elements, which are expressed as a function of N and M. Since the worst case complexity is considered, we assumed that the network is fully connected, i.e., the total number of connections in the network is $K = |\mathcal{A}| + |\mathcal{B}|$, where $|\mathcal{A}| = MN$, and $|\mathcal{B}| = M(M-1)/2$.

It should be pointed out that the algorithms mentioned here for solving the source localization are not uniquely defined in a clear primal or dual form; thus, we can interpret them in the form that is more suitable for the solver [35]. For example, if we interpret the cooperative localization problem defined in [12] in dual form, we get $((M+2)(M+3)/2) + K$ variables, which correspond to $((M+2)(M+3)/2) + K$ equality constraints in the primal form. In contrast, if we interpret the same problem in the primal form, we get $4K + 3$ equality constraints, corresponding to the same number of variables in the dual form. While performing the simulations, we have experienced that the latter interpretation is computationally more efficient; thus, the complexity analysis is performed based on the primal form representation only.

To provide a more complete overview of the algorithm's performance to the reader, we present the complexity results together with the simulations results in the following section.

V. Performance Results

Here, we present a set of performance results to compare the proposed approaches with the existing approaches, for both noncooperative and cooperative localization with known and unknown P_T. All of the presented algorithms were solved by using the MATLAB package `CVX` [21], where the solver is SeDuMi [36].

A. Noncooperative Localization

To generate the RSS measurements, the propagation model (1) is used. Extensive simulations have been carried out to compare the performance of the proposed methods in Section II with that of the existing methods for the cases of known and unknown source transmit power. Unless stated otherwise, in all simulations presented here, the number of Monte Carlo runs is $M_c = 10\,000$, the PLE is $\gamma = 3$, the reference distance $d_0 = 1$ m, and the path loss $L_0 = 40$ dB. The anchor nodes are uniformly distributed at a circle with the center at the origin, and the radius of the circle $r = 20$ m. A source is randomly distributed inside the square region $\{(x,y) | -B \leq x \leq B, -B \leq y \leq B\}$, where B will be defined below, and it is able to communicate with all anchors. The performance metric is the root mean square error (RMSE), which is defined as

$$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{M_c} \frac{\|\boldsymbol{x}_i - \widehat{\boldsymbol{x}}_i\|^2}{M_c}}$$

where $\widehat{\boldsymbol{x}}_i$ denotes the estimate of the true source location, i.e., $\boldsymbol{x}_i$, in the ith Monte Carlo run for the specific noise realization. The Cramér–Rao lower bound (CRB) on the RMSE of any unbiased estimator is employed as a performance benchmark (see Appendix A for more details).

1) Known P_T: Table I gives an overview of the considered algorithms in this section, together with their complexities. In [12] and [13], the authors have considered both indoor and outdoor localization scenarios. Our simulation results show that, for the chosen scenario, the proposed approach exhibits just a marginal gain when compared with the existing approaches. Thus, for the noncooperative scenario when P_T is known, we focus on indoor localization only.

Indoor localization: In Appendix B, we give more details about the indoor propagation model. The simulation results

TABLE I
SUMMARY OF THE CONSIDERED ALGORITHMS IN SECTION V-A1

Algorithm	Description	Complexity
$\mathrm{SDP}_{\mathrm{RSS}}$	The SDP estimator in [12]	$\mathcal{O}\left(N^{4.5}\right)$
WLS-1	The WLS estimator in [13]	$\mathcal{O}\left(N\right)$
SOCP1	The proposed SOCP estimator in (6)	$\mathcal{O}\left(N^{3.5}\right)$

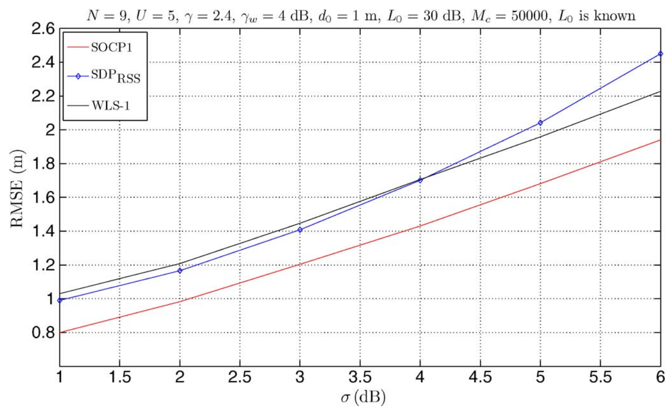

Fig. 1. Simulation results for noncooperative localization in indoor environment when L_0 is known: RMSE versus σ when $N = 9$, $U = 5$ dB, $\gamma = 2.4$, $\gamma_w = 4$ dB, $d_0 = 1$ m, $L_0 = 30$ dB, and $M_c = 50\,000$.

TABLE II
SUMMARY OF THE CONSIDERED ALGORITHMS IN SECTION V-A2

Algorithm	Description	Complexity
WLS-2	The WLS estimator in [16]	$\mathcal{O}\left(N\right)$
SDP-URSS	The SDP estimator in [18]	$\mathcal{O}\left(N^{3.5}\right)$
SOCP2	The proposed SOCP estimator in II-B1	$2 \cdot \mathcal{O}\left(N^{3.5}\right)$

for indoor localization are presented in Fig. 1. The scenario described in [13] is used to execute the comparison of the performances. Fig. 1 clearly demonstrates the superiority of the proposed approach over the existing approaches for the whole range of σ. To illustrate this fact, consider the cases when $\sigma =$ 1 dB and $\sigma = 6$ dB. For the former case, "SOCP1" shows a gain of approximately 0.2 m when compared with the existing approaches. For the latter case, "SOCP1" outperforms "WLS-1," and "$\mathrm{SDP}_{\mathrm{RSS}}$" approaches by 0.3 and 0.5 m, respectively. In summary, the proposed approach outperforms the existing approaches in terms of the estimation accuracy with an average error reduction of about 15%, whereas in terms of the computational complexity, it represents a solid alternative.

2) Unknown P_T: Table II gives an overview of the considered algorithms in this section, together with their complexities. Fig. 2 shows the comparison of the RMSE versus N for $\sigma =$ 5 dB, $B = 15$ m and $r = 20$ m. In Fig. 2, we observe that the estimation error decreases as N is increased, as expected. Furthermore, we observe the superior RMSE performance of the "SOCP2" approach for all chosen N. Fig. 2 also shows that the performance margin between the proposed and the existing approaches grows as N is increased, e.g., for $N = 24$, "SOCP2" outperforms the existing approaches by more than 0.5 m, which is a relatively large gain considering that when $N = 24$, the amount of information gathered in the network is significant. This can be explained, to some extent, by the fact that our approach has somewhat higher computational complexity than the existing approaches and the use of the proposed

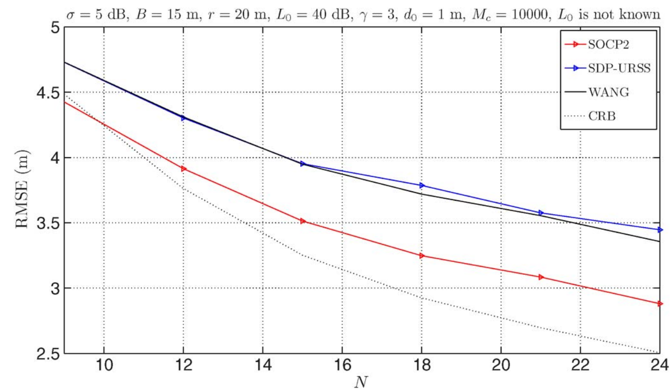

Fig. 2. Simulation results for noncooperative localization when L_0 is not known: RMSE versus N when $\sigma = 5$ dB, $B = 15$ m, $r = 20$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$.

TABLE III
L_0 ESTIMATION ANALYSIS FOR THE "SOCP2" APPROACH

N	After II - step		After III - step	
	$\mathbf{RMSE}_{L_0}$	$\mathbf{Bias}_{L_0}$	$\mathbf{RMSE}_{L_0}$	$\mathbf{Bias}_{L_0}$
3	4.0639	1.8360	3.0967	0.4366
6	2.2483	0.3557	2.0541	0.0642
9	1.7591	0.1344	1.6997	0.0260
12	1.4582	0.0450	1.4476	0.0116
15	1.3135	0.0500	1.3029	0.0238
18	1.1836	0.0432	1.1774	0.0288
21	1.1097	0.0084	1.1072	0.0004

TABLE IV
SUMMARY OF THE CONSIDERED ALGORITHMS IN SECTION V-A.3

Algorithm	Description	Complexity
WLS-3	The WLS estimator in [16]	$K_{max} \cdot \mathcal{O}\left(N\right)$
SOCP3	The proposed SOCP estimator in II-C1	$K_{max} \cdot \mathcal{O}\left(N^{3.5}\right)$

three-step procedure. When N increases, we can obtain a better estimate of P_T, i.e., L_0, in the first step of our procedure, which then allows us to achieve high estimation accuracy in the second step with respect to (w.r.t.) $\boldsymbol{x}$ (see Table III for more details). As shown, after the second step of our procedure, we obtain an excellent estimate of L_0, which we then employ in the third step. From Table III, we observe that as N is increased, the estimation of L_0 is improved. Although the estimation of L_0 in the second step is not perfect, the simulation results in Fig. 2 confirm the robustness of our SOCP approach to the imperfect knowledge of L_0.

In Fig. 2, we can see that the new approach is biased for small N. This is not surprising since it is known that RSS-based algorithms are generally biased [2]. For the sake of completeness, we also compared the considered approaches in terms of bias. The bias is defined as Bias $= \|(1/M_c)\sum_{i=1}^{M_c}(\boldsymbol{x}_i - \hat{\boldsymbol{x}}_i)\|_1$, where $\| \bullet \|_1$ represents the l_1 norm. In our simulations, we have observed that all approaches are slightly biased. However, the bias error is less than 0.14 m for all estimators, which is a relatively small error in the case when the RMSE error is of the order of a few meters.

3) Unknown P_T and γ: Table IV gives an overview of the considered algorithms in this section, together with their complexities. The results in [38] imply that better estimation accuracy w.r.t. $\boldsymbol{x}$ is attained by choosing the initial value of the PLE, i.e., $\hat{\gamma}^0$, to be greater than the true value of γ. This

TABLE V
UNKNOWN PARAMETER ESTIMATION ANALYSIS FOR A RANDOM CHOICE OF $\hat{\gamma}_0 \in [\gamma_{\min}, \gamma_{\max}]$

N	$\hat{\gamma}_0 \in [\gamma_{min}, \gamma_{max}]$						
	RMSE$_{\boldsymbol{x}}$ (m)	**Bias$_{\boldsymbol{x}}$** (m)	**RMSE$_{L_0}$** (dB)	**Bias$_{L_0}$** (dB)	**RMSE$_\gamma$**	**Bias$_\gamma$**	**Av. iter.**
3	9.7700	0.0401	8.6155	0.1203	0.5715	0.0152	18.9713
6	5.6250	0.0531	7.8665	1.2038	0.5742	0.1042	11.9949
9	4.7049	0.1396	7.4599	0.7406	0.5540	0.0624	10.2199
12	4.2493	0.0366	7.3174	0.7629	0.5500	0.0634	9.1817
15	3.8891	0.0207	7.2133	0.6421	0.5448	0.0531	8.4968
18	3.6239	0.0422	7.1992	0.6195	0.5455	0.0492	7.9642
21	3.4266	0.0181	7.1175	0.4674	0.5388	0.0390	7.5611

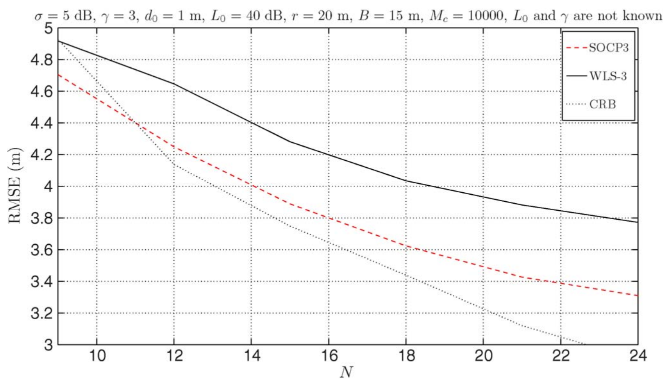

Fig. 3. Simulation results for noncooperative localization when L_0 and γ are not known: RMSE versus N when $\sigma = 5$ dB, $B = 15$ m, $r = 20$ m, $L_0 = 40$ dB, $d_0 = 1$ m, $K_{\max} = 30$, $\gamma_{\min} = 2$, $\gamma_{\max} = 4$, $\epsilon = 10^{-3}$, and $M_c = 10\,000$.

motivated us to investigate the influence of the choice of $\hat{\gamma}^0$ on the estimation accuracy of the remaining unknown parameters. Our analysis showed that the influence of the choice of $\hat{\gamma}^0$ on the estimation accuracy of the source positions is significant. We present some of these results in Table V. Note that the average number of iterations for an algorithm to converge is denoted by "Av. iter." Generally, the best estimation accuracy w.r.t. $\boldsymbol{x}$ is achieved for $\hat{\gamma}^0 = \gamma_{\max}$. However, when the key requirement is to estimate L_0 or γ, then a random choice of $\hat{\gamma}^0 \in [\gamma_{\min}, \gamma_{\max}]$ is by far the best choice. The price to pay for this choice of $\hat{\gamma}^0$ is somewhat slower convergence, in comparison to the choice $\hat{\gamma}^0 = \gamma_{\max}$.

Fig. 3 shows the comparison of the RMSE versus N for $\sigma = 5$ dB, $B = 15$ m, and $r = 20$ m. In this figure, we represent a curve for the proposed approach for a random choice of $\hat{\gamma}^0 \in [\gamma_{\min}, \gamma_{\max}]$. Fig. 3 confirms the efficiency of the iterative procedure. It shows that the proposed approach is robust to the imperfect knowledge of the PLE, since it suffers only a small deterioration of the performance when both P_T and PLE are not known, compared with the case when only P_T is unknown. Furthermore, Fig. 3 shows the superior RMSE performance of the proposed approach for all chosen N. We observe that the performance margin between "SOCP3" and "WLS-3" increases as N increases. To illustrate this fact, consider $N = 9$ and $N = 24$. In the former case, the "SOCP3" approach outperforms "WLS-3" by roughly 0.3 m, whereas in the latter case, it shows a gain of roughly 0.5 m. In Fig. 3, it is shown that "SOCP3" achieves the value RMSE $= 4$ m with four anchors less than "WLS-3," which can reduce the cost of the network implementation in practice. Note that this gain comes at a cost of increased complexity.

TABLE VI
SUMMARY OF THE CONSIDERED ALGORITHMS IN SECTION V-B1

Algorithm	**Description**	**Complexity**
SDP$_{\text{RSS}}$	The SDP estimator in [12]	$\mathcal{O}\left(M^{0.5}\left(48M^4\left(N+\frac{M}{2}\right)^4\right)\right)$
SD/SOCP-1[4]	The mixed SD/SOCP estimator in [13]	$2\cdot\mathcal{O}\left(M^{0.5}\left(M^4\left(N+\frac{M}{2}\right)^2\right)\right)$
SDP1	The proposed SDP estimator in (19)	$\mathcal{O}\left(\sqrt{2}M^{0.5}\left(4M^4\left(N+\frac{M}{2}\right)^2\right)\right)$

B. Cooperative Localization

This section presents the simulation results for the cooperative localization problem. The performance of the proposed approaches in Sections III-A and B will be compared with the existing algorithms for the cases of known and unknown P_T. In all simulation results presented here, the number of Monte Carlo runs is $M_c = 10\,000$, the PLE is $\gamma = 3$, the reference distance $d_0 = 1$ m, and the path loss $L_0 = 40$ dB. Unless stated otherwise, the anchor nodes are fixed at the positions (B, B), $(0, B)$, $(-B, B)$, $(-B, 0)$, $(-B, -B)$, $(0, -B)$, $(B, -B)$, $(B, 0)$, and (0,0), where B will be defined below. Source nodes are randomly deployed inside the convex hull of the anchor nodes, and due to the limited communication range, only some nodes can directly connect to the anchors. The performance metric is the normalized RMSE (NRMSE), which is defined as

$$\text{NRMSE} = \sqrt{\frac{1}{M}\sum_{i=1}^{M_c}\sum_{j=1}^{M}\frac{\|\boldsymbol{x}_{ij} - \hat{\boldsymbol{x}}_{ij}\|}{M_c}}$$

where $\widehat{\boldsymbol{x}}_{ij}$ denotes the estimate of the true location of the jth source, $\boldsymbol{x}_{ij}$, in the ith Monte Carlo run for the specific noise realization.

1) Known P_T: Table VI gives an overview of the considered algorithms in this section, together with their complexities. We observe from Table VI that the proposed SDP approach has somewhat higher computational complexity than the "SD/SOCP-1" approach and lower complexity than the "SDP$_{\text{RSS}}$" approach. Although somewhat more complex than the "SD/SOCP-1" approach, the proposed approach has the lowest running time, as one can see from Table VII(a). This is due to the fact that for cooperative localization, when P_T is known, we solve only one SDP, whereas "SD/SOCP-1" involves solving two SDP problems.

Fig. 4 shows the comparison of the NRMSE versus M, for $N = 9$, $R = 6$ m, $\sigma = 5$ dB, and $B = 15$ m. We observe

TABLE VII
AVERAGE RUNNING TIME OF THE CONSIDERED ALGORITHMS FOR THE COOPERATIVE LOCALIZATION. $N = 8$, $M = 20$, AND $R = 6$ m. CPU: INTEL® CORE™ 7-363QM 2.40 GHz. (a) P_T IS KNOWN. (b) P_T IS NOT KNOWN

(a)

Algorithm	**Time** (s)
SDP_{RSS}	1.5539
SD/SOCP-1	0.9680
SDP1	0.8691

(b)

Algorithm	**Time** (s)
SD/SOCP-2	1.0898
SDP-URSS	0.7232
SDP2	1.7949

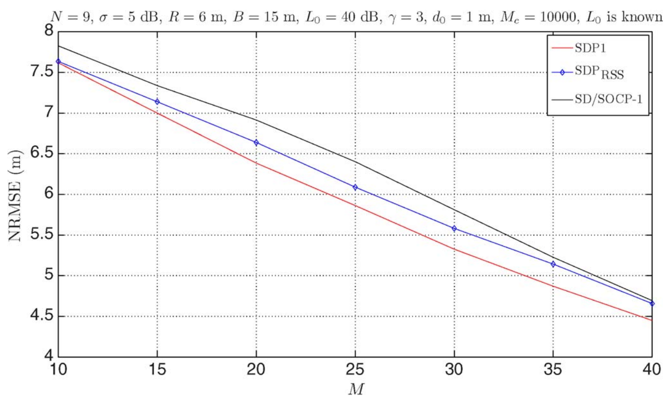

Fig. 4. Simulation results for cooperative localization when L_0 is known: NRMSE versus M when $N = 9$, $\sigma = 5$ dB, $B = 15$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$.

TABLE VIII
SUMMARY OF THE CONSIDERED ALGORITHMS IN SECTION V-B2

Algo-rithm	**Description**	**Complexity**
SD/SOCP-2	The mixed SD/SOCP estimator in [16]	$2 \cdot \mathcal{O}\left(M^{0.5}\left(M^4\left(N + \frac{M}{2}\right)^2\right)\right)$
SDP-URSS	The SDP estimator in [18]	$\mathcal{O}\left(M^{0.5}\left(M^4\left(N + \frac{M}{2}\right)^2\right)\right)$
SDP2	The proposed SDP estimator in III-B1	$2 \cdot \mathcal{O}\left(\sqrt{2}M^{0.5}\left(4M^4\left(N + \frac{M}{2}\right)^2\right)\right)$

that, as M increases, the estimation accuracy of all algorithms improves, which is intuitive since more information is collected inside the network. Furthermore, in Fig. 4, it is shown that the proposed approach outperforms the existing approaches for all values of M, with the biggest improvement for medium-to-high M. The proposed approach outperforms on average "SD/SOCP-1" by roughly 0.5 m, although "SD/SOCP-1" solves two SDP problems. Moreover, the new approach improves the estimation accuracy on average by about 0.25 m in comparison to "SDP_{RSS}," although "SDP_{RSS}" is more computationally complex. In short, albeit "SDP_{RSS}" is more computationally complex than the new approach and "SD/SOCP-1" solves two SDP problems, which increases its execution time, the new approach outperforms both of them for all M.

2) Unknown P_T: Table VIII gives an overview of the considered algorithms in this section, together with their complexities. From Table VIII, it can be seen that the proposed approach has the highest computational complexity. This was confirmed in our simulations (see Table VII(b) for more details).

In Fig. 5, we present one possible network configuration and the estimation accuracy of the source positions accomplished by the "SDP2" approach, for $N = 9$, $M = 40$, $R = 6$ m, $\sigma = 5$ dB, and $B = 15$ m. In Fig. 5, it is shown that better estimation accuracy is achieved for source nodes with higher number of connections (neighbors), as anticipated.

Fig. 6 shows the NRMSE performance of the considered approaches for different M, when $N = 9$, $R = 6$ m, $\sigma = 5$ dB, and $B = 15$ m. We can see that the estimation accuracy improves as M is increased, as expected. Fig. 6 confirms the superiority of our approach, since it outperforms the existing approaches for all choices of M, with an average gain of about 0.5 m. When the information gathered by the network is not enough (low M), our approach outperforms "SD/SOCP-2" and "SDP-URSS" by roughly 2 and 1 m, respectively. As M increases, the performance margin between all approaches decreases, as anticipated, since the information inside the network becomes sufficient to allow good performance for all estimators. Finally, in Figs. 4 and 6, it is shown that the new approach suffers only a marginal deterioration in the performance for the scenario where P_T is not known.

Fig. 7 shows the NRMSE performance of the considered approaches for different R, when $N = 9$, $M = 15$, $\sigma = 5$ dB, and $B = 15$ m. As anticipated, the estimation error decreases when R is increased. In Fig. 7, it is clear that the proposed approach outperforms the existing approaches for all choices of R, with an average gain of more than 1 and 0.5 m compared with "SD/SOCP-2" and "SDP-URSS," respectively. An important practical scenario would be the case where R is chosen to be low, due to the need to preserve low energy consumption in the network. In this case, our approach reduces the estimation error by about 2 and 0.5 m, compared with "SD/SOCP-2" and "SDP-URSS," respectively. Intuitively, as R is increased, all methods are expected to perform good, since the information gathered by the network becomes sufficient enough. However, one can see in Fig. 7 that the performance gains between the proposed approach and "SDP-URSS" increases with R; for $R = 10$ m, the new approach outperforms "SDP-URSS" by almost 1 m. This result further confirms the superiority of the proposed approach over "SDP-URSS."

Fig. 8 shows the NRMSE performance of the considered approaches for different N, when $M = 15$, $R = 6$ m, $\sigma = 5$ dB, and $B = 15$ m. Both anchor and source nodes were randomly positioned inside a square region of length $2B$. This scenario is of particular practical importance because a common requirement for a network is to be flexible and adaptable to different layouts. Fig. 8 shows that the estimation accuracy increases as N is increased, as predicted. One can see that our approach outperforms the existing approaches for all choices of N, with an average gain of about 1 and 0.5 m, compared with "SD/SOCP-2" and "SDP-URSS," respectively. Having less anchor nodes in the network might reduce its implementation costs, since, e.g., they might be equipped with a GPS to determine their own positions. Fig. 8 shows that the proposed approach needs, on average, one or two less anchor nodes to achieve the same estimation accuracy as the state-of-the-art approaches.[4]

[4]The CRB was not presented in Figs. 4–8, since the Fisher information matrix (FIM) is singular for the chosen scenarios.

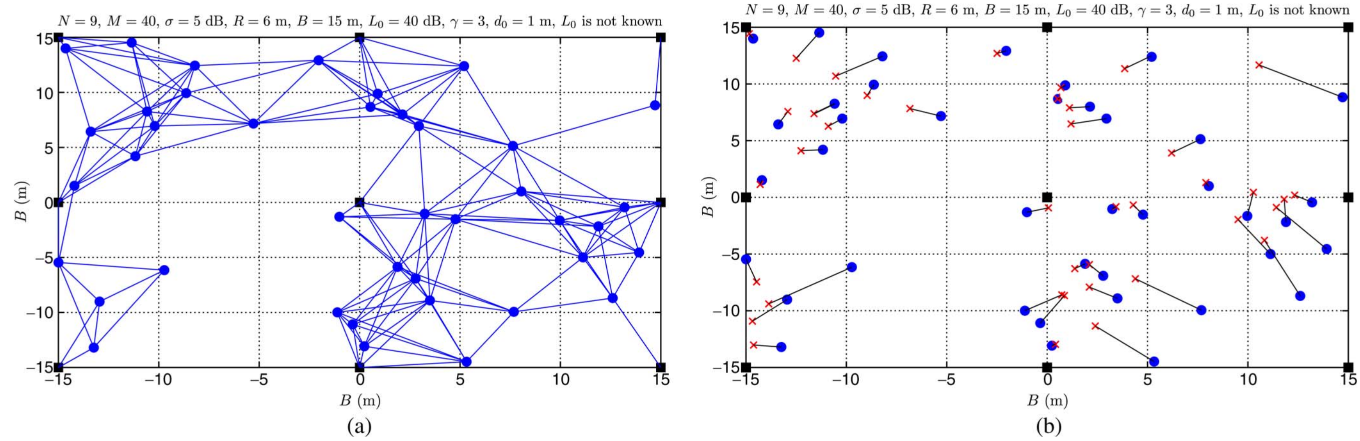

Fig. 5. Example of (a) a network configuration and (b) estimation accuracy results for the "SDP2" approach. Black squares represent the locations of the anchor nodes, blue circles represent the true locations of the source nodes, and red symbols "X" represent the estimated source locations.

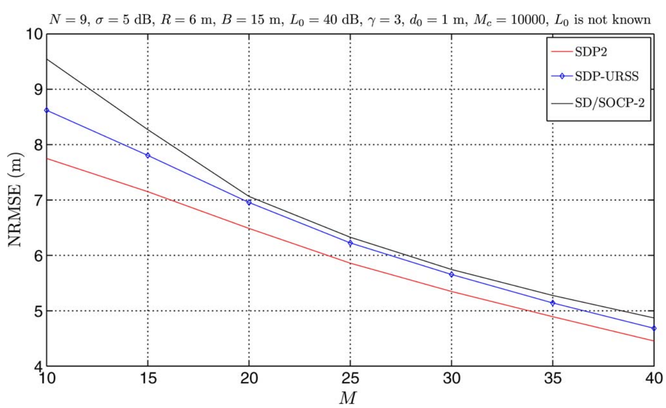

Fig. 6. Simulation results for cooperative localization when L_0 is not known: NRMSE versus M when $N = 9$, $R = 6$ m, $\sigma = 5$ dB, $B = 15$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$.

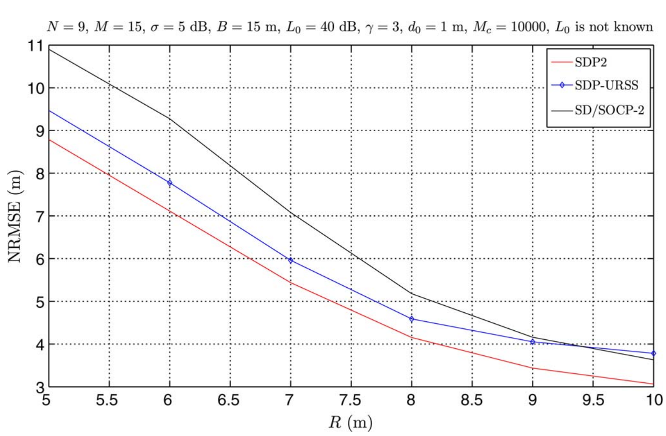

Fig. 7. Simulation results for cooperative localization when L_0 is not known: NRMSE versus R when $N = 9$, $M = 15$, $\sigma = 5$ dB, $B = 15$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$.

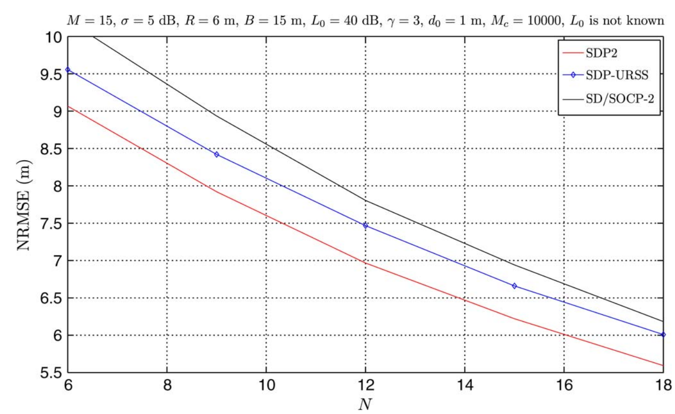

Fig. 8. Simulation results for cooperative localization when L_0 is not known: NRMSE versus N when $M = 15$, $R = 6$ m, $\sigma = 5$ dB, $B = 15$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$. Anchors and sources are randomly deployed inside the square region of length $2B$.

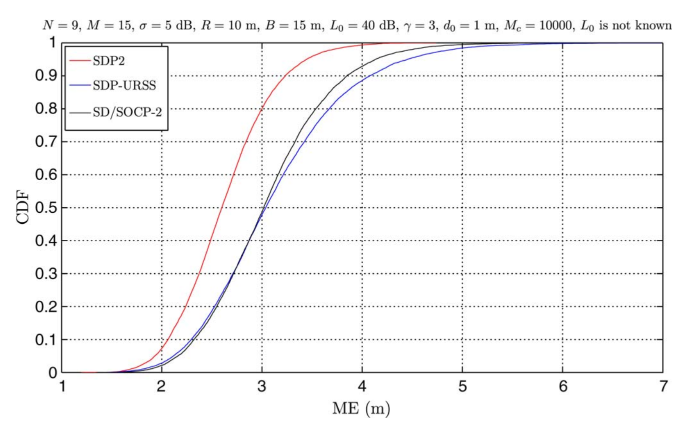

Fig. 9. Simulation results for cooperative localization when L_0 is not known: CDFs of mean errors in source position estimation when $N = 9$, $M = 15$, $R = 10$ m, $\sigma = 5$ dB, $B = 15$ m, $L_0 = 40$ dB, $\gamma = 3$, $d_0 = 1$ m, and $M_c = 10\,000$.

Fig. 9 shows the CDF comparison of the mean error (ME) in the source position estimation of the considered approaches, for $N = 9$, $M = 15$, $R = 10$ m, $\sigma = 5$ dB, and $B = 15$ m. ME is defined as $\mathrm{ME} = \sum_{i=1}^{M}(\|\boldsymbol{x}_{ij} - \hat{\boldsymbol{x}}_{ij}\|/M)$ (m), for $j = 1, \ldots, M_c$, where $\boldsymbol{x}_{i,j}$ and $\hat{\boldsymbol{x}}_{i,j}$ denote the true and the estimated source positions of the ith source node in the jth Monte Carlo run, respectively. Fig. 9 shows that the proposed approach outperforms the existing approaches for all range of ME, improving the estimation accuracy by more than 0.5 m, on average. We can see that the new method achieves $\mathrm{ME} \leq 3$ m in 80% of the cases, whereas the existing methods attain the same value of ME in less than 50% of the cases.

VI. Conclusion

In this paper, we have addressed the RSS-based source localization problem. Both noncooperative and cooperative localization problems were investigated for both cases of known and unknown source transmit power, i.e., P_T.

In the case of the noncooperative localization when P_T is known, we proposed the novel SOCP-based approach, which has an excellent tradeoff between the performance and the computational complexity, when compared with the existing approaches. For the case where P_T is not known, we introduced the simple three-step procedure based on the SOCP relaxation. The simulation results show that the proposed approach provides not only an excellent estimation of the source positions but also an excellent estimation of P_T. This motivated us to exploit the estimate of P_T and solve another SOCP problem as if P_T was known. The price we have to pay for applying this procedure was solving the problem twice. However, the simulation results confirm its effectiveness and show a remarkable improvement of the estimation accuracy. A gain of more than 15% was achieved, when comparing with existing approaches, for the case where N is high. We concluded the noncooperative localization problem by investigating the case where both P_T and PLE are simultaneously unknown. By applying the standard iterative procedure, we showed that our method efficiently solves the most challenging scenario of the RSS localization problem and outperforms the existing method in terms of the estimation accuracy; the biggest gain is obtained for high N, where an improvement in the localization accuracy of about 15% is attained. Moreover, we have shown that our approach can be used to solve the noncooperative localization problem in indoor environments. The new approach reduces the estimation error by more than 15% when compared with state-of-the-art approaches.

In the case of the cooperative localization when P_T is known, we proposed the novel SDP-based approach. In huge contrast to the existing SDP approaches that consider the cooperative localization problem, in which the unknown variables are treated as a matrix, here, we consider them as a vector. This approach implies a slight increase in complexity in comparison to state-of-the-art methods. However, the performance evaluation in Section V justifies the use of such an approach. For the case where P_T is known, the proposed approach outperforms the existing approaches for all choices of M with the biggest margin for medium-to-high M. In the case where P_T is assumed to be not known, the simple three-step procedure based on SDP relaxation is applied. We have investigated the influence of different M and N, as well as different R on the estimation accuracy. For all the scenarios presented in this paper, the new approach outperforms the state-of-the-art approaches with an increase in accuracy between 15% and 20% on average. Furthermore, the simulation results show that our approach achieves ME $\leq$ 3 m in 80% of the cases, whereas the existing approaches accomplish the same accuracy in less than 50% of the cases.

Appendix A
Cramer–Rao Bound Derivation

CRB provides a lower bound on the variance of any unbiased estimator, which means that it is physically impossible to find an unbiased estimator whose variance is less than the bound. CRB offers us a benchmark against which we can compare the performance of any unbiased estimator. If the estimator attains the bound for all values of the unknown parameters, we say that such estimator is the minimum variance unbiased estimator [19].

Let $\boldsymbol{\theta} = [\boldsymbol{x}_k^T, L_0, \gamma]^T$, $k = 1, \ldots, M$, denote the $2M+2$ vector of all unknown parameters. According to [19], the variance of any unbiased estimator is lower bounded by $\mathrm{var}(\hat{\boldsymbol{\theta}}_i) \geq [\boldsymbol{J}^{-1}(\boldsymbol{\theta})]_{ii}$, where $\boldsymbol{J}(\boldsymbol{\theta})$ is the $(2M+2) \times (2M+2)$ FIM. The elements of the FIM are defined as $[\boldsymbol{J}(\boldsymbol{\theta})]_{i,j} = -E[\partial^2 \ln p(\boldsymbol{L}|\boldsymbol{\theta}) / \partial \boldsymbol{\theta}_i \partial \boldsymbol{\theta}_j]$, where $i, j = 1, \ldots, (2M+2)$, and $p(\boldsymbol{L}|\boldsymbol{\theta})$ is the joint conditional probability density function of the observation vector $\boldsymbol{L} = [L_1, \ldots, L_K]$, given $\boldsymbol{\theta}$.

The FIM is computed as

$$\boldsymbol{J}(\boldsymbol{\theta}) = \frac{1}{\sigma^2} \sum_{(i,j):(i,j)\in\mathcal{A}} \boldsymbol{f}_{ij} \boldsymbol{f}_{ij}^T + \frac{1}{\sigma^2} \sum_{(i,k):(i,k)\in\mathcal{B}} \boldsymbol{f}_{ik} \boldsymbol{f}_{ik}^T \quad (25)$$

where

$$\boldsymbol{f}_{ij} = \boldsymbol{\rho} + 10\boldsymbol{g} \log_{10} \frac{\left\|\boldsymbol{D}_i^T\boldsymbol{\theta} - \boldsymbol{s}_j\right\|}{d_0} + \frac{10\boldsymbol{g}^T\boldsymbol{\theta} d_0}{\ln(10)} \frac{\boldsymbol{D}_i \boldsymbol{D}_i^T \boldsymbol{\theta} - \boldsymbol{D}_i \boldsymbol{s}_j}{\left\|\boldsymbol{D}_i^T \boldsymbol{\theta} - \boldsymbol{s}_j\right\|^2},$$

$$\boldsymbol{f}_{ik} = \boldsymbol{\rho} + 10\boldsymbol{g} \log_{10} \frac{\left\|\boldsymbol{D}_i^T\boldsymbol{\theta} - \boldsymbol{D}_k^T\boldsymbol{\theta}\right\|}{d_0} + \frac{10\boldsymbol{g}^T\boldsymbol{\theta} d_0}{\ln(10)} \frac{\boldsymbol{D}_i \boldsymbol{D}_i^T \boldsymbol{\theta} - \boldsymbol{D}_i \boldsymbol{D}_k^T \boldsymbol{\theta} - \boldsymbol{D}_k \boldsymbol{D}_i^T + \boldsymbol{D}_k \boldsymbol{D}_k^T \boldsymbol{\theta}}{\left\|\boldsymbol{D}_i^T\boldsymbol{\theta} - \boldsymbol{D}_k^T\boldsymbol{\theta}\right\|^2}$$

and $\boldsymbol{\rho} = [\boldsymbol{0}_{2M\times 1}; 1; 0]$, $\boldsymbol{g} = [\boldsymbol{0}_{(2M+1)\times 1}; 1]$, $\boldsymbol{D}_i = [\boldsymbol{p}_{2i-1}, \boldsymbol{p}_{2i}]$, where $\boldsymbol{p}_i$ represents the ith column of the identity matrix $\boldsymbol{I}_{2M+2}$.

Therefore, the CRB for the estimate of the source positions is computed as

$$\mathrm{CRB} = \mathrm{tr}\left(\left[\boldsymbol{J}^{-1}(\boldsymbol{\theta})\right]_{1:2M,1:2M}\right) \quad (26)$$

where $[\boldsymbol{M}]_{a:b,c:d}$ represents the submatrix of matrix $\boldsymbol{M}$ composed of the rows a to b and columns c to d of $\boldsymbol{M}$.

Appendix B
Indoor Localization

In practice, the attenuation in indoor environments is superior than that in outdoor environments due to additional deteriorating caused by obstacles (such as walls, floors, and other objects) and multipath fading. Therefore, the propagation model (1) is not suitable for indoor localization, and hence, we adopt a different propagation model [13], i.e.,

$$L_i = L_0 + L_{w,i} + 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0} + v_i, \quad i = 1, \ldots, N \quad (27)$$

where $L_{w,i}$ is the path-loss term that represents the attenuation caused by partitions and multipath fading. Similar to the model in [13], we assume that $L_{w,i} = n_{w,i}\gamma_w + u_i\Pi_w$, where $n_{w,i}$ represents the number of partitions that the signal passes through, γ_w is the partition attenuation factor, and $u_i = U\sin(2\pi t/t_u)$ is a random variable that models the varying indoor environment. The samples of t were drawn from the uniform distribution on the interval $[0, t_u]$. Π_w is the indicator function that indicates whether the signal passes through partitions or not, i.e.,

$$\Pi_w = \begin{cases} 0, & \text{if } n_{w,i} = 0 \\ 1, & \text{otherwise.} \end{cases}$$

Letting $\tilde{v}_i = u_i\Pi_w + v_i$ and $\tilde{L}_{0,i} = L_0 + n_{w,i}\gamma_w$, the following problem formulation is obtained:

$$L_i = \tilde{L}_{0,i} + 10\gamma\log_{10}\frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0} + \tilde{v}_i, \quad i = 1, \ldots, N. \quad (28)$$

In [13], it was shown that (28) can be expressed in the similar form as (1), using some approximations for $\tilde{v}_i$. Applying these approximations, the implementation of the described algorithms for the noncooperative localization when P_T is known is straightforward for the case of indoor localization.

ACKNOWLEDGMENT

The authors would like to thank Dr. J. Löfberg from ETH Zürich, Zürich, Switzerland, and Dr. G. Wang from Ningbo University, Ningbo, China, for a constructive discussion about the conic optimization problems. They would also like to thank the associate editor, Prof. G. Mao, and the anonymous reviewers for their valuable comments and suggestions that improved the quality of this paper.

M. Beko is a collaborative member of INESC-INOV, Instituto Superior Técnico, Universidade de Lisboa, Lisbon, Portugal.

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