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FIF: A fuzzy information fusion algorithm based on multi-criteria decision making

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ABSTRACT

The main goal of information fusion is to combine heterogeneous information to obtain a single composite of potential comparable alternative solutions that can be classified and ranked. The crux of information fusion, which is a type of data fusion, is threefold: (i) data must be comparable and numerical, using some normalization process; (ii) imprecision in data must be taken into consideration; (iii) an appropriate aggregation function to combine values into a single score must be selected.

Recently, computational intelligence concepts and techniques to perform data/information fusion are emerging as suitable tools. Although with a different perspective, another field where much work has also been done for combining heterogeneous information is multi-criteria decision-making. In general, multi-criteria problems are modelled by choosing a set of relevant criteria – usually dealing with heterogeneous data – that have to be aggregated (i.e. fused) to obtain a single rating for each candidate alternative.

In this paper we propose an algorithm for data/information fusion, which includes concepts from multi-criteria decision-making and computational intelligence, specifically, fuzzy multi-criteria decision-making and mixture aggregation operators with weighting functions. The application field of interest for this work is safe spacecraft landing with hazard avoidance; hence two existing hazard maps will be used to illustrate the versatility of the algorithm.

1. Introduction

Data fusion is a wide concept and several definitions have been proposed in the literature (see for example [3,30,26]). In general, we can say that "Data Fusion" is any process of aggregating data from multiple sources into a single composite with higher information quality. It can be viewed from different perspectives by different domains, being the most common branches: image fusion, multi-sensor fusion and information fusion [26]. The common denominator for all three domains is that the different sources must address/represent the same subject (e.g. scene, action, local, event, alternative, etc.) and that they share available techniques from statistical, probabilistic or computational intelligence methods [30]. Data fusion techniques and approaches are widely used in many fields such as robotics, medicine, environment, military applications, financial and so forth (see for example [28,18,19,34,26]). Databases are another important and growing field of application for information fusion [3]. In this work, the application field of interest is hazard map fusion and the corresponding branch is information fusion.

In the context of hazard map fusion, most literature falls on the image-fusion field, specifically in image processing techniques, because the goal is to produce maps with higher quality using image processing methods [11]. In opposition, our method does not include any image processing technique. We look at hazard maps as matrices of numbers that are normalized to represent concepts (information) such as "low slope", "low visibility" or "low fuel" and then this information is fused into a single composite of candidate alternatives, i.e. each cell is a potential alternative solution. Recently, the application of computational intelligence concepts and techniques [13] for performing information fusion are emerging as versatile tools [28,18,20,24,55]. Particularly, the application of fuzzy set theory and neural networks – two components of computational intelligence – to perform information fusion are being put forward [29,28,30,38,60,11]. Further, there are already some interesting applications of fuzzy rule-based systems for hazard mapping fusion [47,51] as well as others using a combination of fuzzy set theory and evolutionary optimization algorithms.

Another field where some work in information fusion has been done – although with a different perspective – is multi-criteria decision making [57,58] and particularly its fuzzy extension. The

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main objective of fuzzy multi-criteria decision making is to rank a set of alternatives by aggregating a set of criteria ratings to obtain a final score per alternative. When viewing/considering hazard maps as fuzzy criteria (e.g. slope, reachability), where each cell is a candidate site (an alternative) to be chosen, we obtain a direct match between alternative and pixel selection, within the two paradigms: fuzzy multi-criteria decision making and information fusion.

In this paper we present an algorithm for intelligent information fusion in uncertain environments, denoted Fuzzy Information Fusion (FIF). The algorithm is presented and its theoretical aspects are discussed using an illustrative example with two hazard maps. The proposed algorithm, denoted FIF (Fuzzy Information Fusion), merges concepts from the computational intelligence domain with multi-criteria decision making. The outcome is a composite hazard map where each pixel is represented by two values: a table index (cell location in the hazard map matrix) and its respective rating value.

In summary, FIF is a general algorithm that can be applied to any problem, as long as the criteria can be modelled as fuzzy sets representing any semantic concept (e.g. "low-slope"). FIF versatility also allows customization and tuning of the chosen parameters for expressing relative importance of criteria as well as using any aggregation function for fusion (if necessary).

In addition, the FIF algorithm results from preliminary work where a dynamic (spatial–temporal) decision model was developed and successfully applied to spacecraft safe landing on planets [5,53]. The main differences between this work and the past research are:

(a) FIF objective is not to select alternatives but to fuse information.
(b) FIF targets fusing information available at time k and it is not a spatial–temporal (dynamic) system incorporating feedback (the feedback requires two aggregation processes).
(c) FIF is not bounded by the two phases of the previous dynamic model (data preparation and decision evaluation at each iteration). On the contrary, it just combines appropriate aspects for information fusion, such as: normalization/fuzzification, filtering mechanism to deal with imprecision and reinforcing criteria weighting.
(d) FIF does not require data reduction on the data preparation step as the dynamic model used.
(e) FIF generalizes the aggregation method (mixture operator with weighting functions) used in the dynamic model to deal with any kind of information (qualitative and quantitative), which includes dealing with imprecise information such as lack of confidence in data or inaccuracies on the values.

The paper is organized as follows. In the next section an overview of the background literature for information fusion is presented. Then, the proposed Fuzzy Information Fusion (FIF) algorithm is presented and discussed with an illustrative application to hazard maps fusion. In particular, in Subsection 3.7 we present a comparative study at a theoretical level. Finally, Section 4 presents some final comments about the algorithm and the conclusions of this work.

2. Background overview

In this section we will contextualize this work by presenting the background concepts and technologies used in the FIF algorithm. We start by providing a brief overview on data fusion to establish the boundaries for information fusion. After, we introduce the main concepts of fuzzy multiple criteria decision-making and the aggregation methods in which the FIF algorithm is based.

2.1. Data fusion

Data fusion is a wide ranging subject that is extensively applied to many different areas such as robotics, image processing and intelligent systems, to name a few [28,45]. Despite the many recent advances in this topic, due to the intrinsic imperfections and diversity of sources and contexts, there is still much to be accomplished to obtain a full-proof data fusion method.

In general, data fusion can be divided in three main categories: multi-sensor fusion, image-fusion and information fusion. Other possible classifications, and their correspondence with the three classes, are: signal fusion (falls in the domain of multi-sensor fusion), pixel and feature fusion (falls into the domain image-fusion) and decision fusion (falls on what we are calling information fusion). Next we briefly discuss each type.

The main objective of multi-sensor fusion – fusion of data provided by sensors – is to integrate data measurements extracted from different sensors and combine them into one representation. In this context most approaches focus on multi-sensor data fusion using statistical methods (Kalman filters) and probabilistic techniques (Bayesian networks) [14,17,30,33,59]. Hybrid strategies combine different multi-sensor fusion techniques taking the advantages of the individual approaches and mitigating their flaws (see for example [38]). Good overviews about this topic are provided in [28,30].

The main objective of image fusion – fusing different images into an improved one – is to reduce uncertainty and redundancy while maximizing relevant information particular to an application by combining different image representations of the same scene [18]. The general approach is to use multiple images provided by different sensors, and combine them to obtain a better understanding of the scene. This understanding can be either in terms of position and geometry, or/and in terms of semantic interpretation. Image fusion algorithms are usually divided into two categories: pixel based and feature-symbolic based [9]. In pixel-based fusion, data is merged on a pixel-by-pixel basis. Feature-based fusion requires extraction and fusion of features from different sources. For example, images from different sources are segmented and fused together. However, feature and symbolic algorithms have not received the same level of attention as pixel-level algorithms. Pixel-level algorithms are the most common and they work either in the spatial domain or in the transformation domain. Although pixel-level fusion is a local operation, transformation domain algorithms create fused images globally. Feature-based algorithms (symbolic) typically segment the images into regions and fuse them using their intrinsic properties [24,39].

There are many suitable algorithms and techniques proposed in the literature to deal with the two levels of image fusion processes, such as: multi-resolution analysis, hierarchical image decomposition, pyramid techniques, wavelet transform, artificial neural networks, biological inspired models, fuzzy rules, Principal Component Analysis (PCA) and so forth [11,17,18,24,39,48]. Good overviews on image fusion algorithms and applications can be seen in [18,39,55].

Regarding information fusion, there is a general agreement that it is a multi-level process of combining different data to produce fused information (see for example [30,56]). Moreover, in information fusion (decision-based) the pre-processed outputs of each single source are combined to create a new interpretation. Usually, in the realm of artificial intelligence (or computational intelligence) information fusion has two main goals [56]: to support decision-making and to improve the understanding of an application domain. Further, there is a tenuous line between image fusion and information fusion; for example, feature and symbolic levels of fusion are sometimes considered image-fusion but they can also be considered as information-based fusion [35,39]. Another focus is on supporting decision-making and encompasses different aims, such as: improve available knowledge; update current information; reach consensus; and improve knowledge discovery by joining data [12,21]. In general, information fusion is used in systems to reduce some type of noise, increase accuracy, summarize information, extract useful information, support decision-making and so forth. Good overviews on information fusion methods and applications can be found in [30,45,56].

Our research falls mainly on decision-making support and the specific objective of this work is to fuse hazard maps, i.e. hazard maps are the criteria, to evaluate a set of candidate alternatives. We also follow the view of [45], where the process of combining several numerical values into a single representative is well performed with aggregation functions (so-called operators).

The most traditional and well-known frameworks for information fusion are based on statistical methods (e.g. Kalman filters, optimal theory, distance methods) and probabilistic techniques (e.g. Bayesian networks, Evidence theory) [29,30,11,17,33,59]. Statistical methods focus on minimizing errors between actual and predicted values, whereas probabilistic methods rely on using weighting factors based on how accurate the data is. Interesting applications of these techniques for fusion and hazard avoidance safe landing are [14,15,27,31,43,48,52].

Recently, many other computational intelligence approaches for information fusion have been proposed, particularly using Dempster–Shafer evidential theory, Fuzzy set theory, Random Set theory, decision/aggregation methods, knowledge-based methods and neural networks (see for example, [29,25,30,45,56,62,4,11]. Some interesting applications for safe landing using these novel approaches are [5,10,23,24,36,47]. In Table 2 (Section 3.7) we present a comparison of the main computational intelligence approaches proposed for data/information fusion, specifically those dealing with Hazard Detection and Avoidance (HDA).

In summary, information fusion methods are crucial for obtaining a coherent and uniform view to support decision makers and the correspondent decision-making process. Further, we must ensure that data is comparable and consistent to ensure a suitable outcome from the information fusion.

2.2. Fuzzy multi-criteria decision making and aggregation operators

In this section we present some background about fuzzy multi-criteria decision making concepts and also aggregation operators because both topics are at the core model of the proposed algorithm aggregation operators.

Decision making and particularly fuzzy multi-criteria decision making may be characterized as a process of choosing or selecting 'sufficiently good' solution(s), from a set of potential solutions, to attain a goal or goals [40]. Decision making processes are difficult due to the existence of doubt, conflict and uncertainty as to the outcomes or selection criteria. One way of handling the intrinsic uncertainty in decision making – and the one followed in this work – is fuzzy multi-criteria decision making [40,58]).

The main multi-criteria decision making steps are:

(a) Define the domain of the problem.
(b) Identify the set of criteria (e.g. hazard maps).
(c) Generate/identify the alternatives (candidate solutions).
(d) Define the weights/importance for each criterion.
(e) Aggregate/fuse the criteria scores for each alternative to obtain a single evaluation/rating.
(f) Rank the alternatives to select the "best" alternative.

Table 1
Semantic weights and corresponding values for alpha and beta.

α Parameter (criteria importance)	β Parameter (slope decrease)
• Very important (VI = 1); • Important ($I = 0.8$); • Average importance ($A = 0.6$); • Low importance ($L = 0.3$); • Very low importance (VL = 0.1); • Ignored ($Ig = 0.0$).	• High ($H = 1$); • Medium ($M = 0.67$); • Low ($L = 0.33$); • Null ($N = 0.0$).

Table 2
Main characteristics of recent computational intelligence approaches to hazard detection and avoidance data fusion.

Method	Pros.	Cons.	*Applicability in HDA*	Refs.
DS – evidential Theory	– Good for fusing heterogeneous sensor data with missing and noisy Data – Theoretically sound	– Difficult to define the belief and plausibility measures – Needs application domain knowledge for data representation	Medium/ low	[30,47,49,50]
Fuzzy Set Theory	– High semantic interpretation – Good for heterogeneous data fusion because of intelligent data representation/normalization capability – Good for handling uncertainty and imprecise information – *very versatile for integrating complementary paradigms (knowledge-based and multi-criteria) –Hybrid systems*	– Needs application domain knowledge for data representation – For rule-based paradigms it is context dependent and not easily adaptable	High	[10,22,36,47,48,51]
Neural Networks	– Good for heterogeneous data fusion – Universal function approximator – Good for fusing multiple sensor data with missing and noisy data	– Training requires a good and sufficient sampling set – Non-adaptable without retraining (not applicable to unknown scenarios) – Time consuming training	Medium	[8,16,24]
Evolutionary computing and swarm intelligence	– High computational efficiency, and possible parallelization capability – Accepts linear and non-linear parameters – Several algorithms available, e.g. PSO, GA, Tabu	– Near optimal solution – Difficult to parameterize and does not allow representing uncertainty – Requires difficult initial parameter tuning	Medium/ low	[5,46,54,53]

Ranking the alternatives (step f), is out of scope of hazard map fusion because it implies a final choice/decision and here we are only focused on fusing information.

The main advantages of using concepts and methods from the Fuzzy Multiple Criteria Decision Making (FMCDM) arena are that it automatically provides a standard and intelligent normalization process [36] and also provides access to a panoply of specialized aggregation methods, ranging from weighted averaging, synergetic, compensatory to reinforcement ones [45,1,41,43,56].

Another strong motivation for using a fuzzy multi-criteria decision making paradigm in information fusion of hazard maps is its ability to transform data into numerical and comparable membership functions representing the hazard maps constraints. This process is usually called fuzzification [44] and in our algorithm it is used for data normalization. For information fusion this is of primordial importance to enable fusing heterogeneous data in uncertain environments. Another important motivation for using this paradigm is the availability of specialized reinforcement aggregation methods [41] which can ensure an "intelligent" fusion by penalizing or rewarding the fused values according to their aggregated solution values.

Many aggregation methods are proposed in this domain, ranging from scoring and outranking methods, trade-off schemes, distance-based methods, utility functions, probabilistic methods, fuzzy aggregation operators and other iterative methods (good surveys can be seen in [1,7,56,57]. The majority of methods use "weighted averaging" aggregation functions, of varying degrees of complexity to fuse the information and obtain a final score. Next, we present some more details about aggregation operators.

2.2.1. Aggregation operators

Aggregation operators have been extensively studied in the literature and their usage in fuzzy multi-criteria problems is widely spread (see for example [45,2,56,6]. The choice of an appropriate aggregation operator [2] is a major issue in any information fusion process and it should be carefully considered when addressing any type of information aggregation.

The most well-known classical aggregation operators (methods) are [57,58,6]: max–min methods, generalized mean methods (e.g. weighted sum and weighted product), outranking methods (e.g. conjunctive method), distance based methods (e.g. TOPSIS, compromised ratio), and the pairwise comparison methods (e.g. AHP, ELETRE). In general, aggregation functions (or operators) are extensively used in many domains from computer science to economics and biology [56]. Nowadays they are basically a field in itself with a large panoply of aggregation operators available (see good overviews in [1,45,56,6].

In this paper we focus on generalized mixture operators – a class of mean (additive) operators – using weighting functions that penalize low degrees of criteria performance and reward high criteria performance [37,43]. In this case, instead of assigning single values to the weights, these are represented by a function that will depend on the criteria satisfaction [42]. This aggregation method extends weighted averaging, which is a particular case of generalized mixture operator. The motivation for this choice of operator was its versatility for expressing the imprecision in the relative importance of criteria and also on allowing the addition (extension) of a filter to tackle the lack of confidence and the inaccuracy of the input data (e.g. hazard maps).

The mathematical representation of the mixture operator is:

$$W_i(x_i) = \sum_{i=1}^{n} w_j(x) * x_{ij}$$

where W_i is the aggregation value for the alternative A_i, $x_i = (x_{i1}, \ldots, x_{in})$ and x_{ij} is the satisfaction value of alternative A_i towards criterion C_j, $w_j(x) = \frac{f_j(x_{ij})}{\sum_{j=1}^{n} f_j(x_{ij})}$; where $j = 1, \ldots, n$ are the weighting functions.

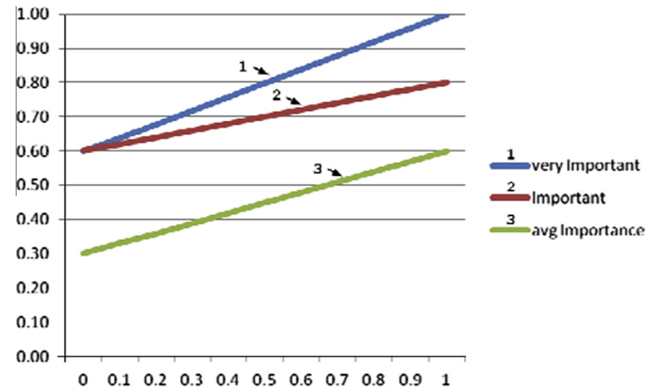

Fig. 1. Examples of linear weighting functions, $l(x)$,for establishing relative importance of criteria VI, I and AVG using slopes (medium, low, high).

There are two families for this operator: one associated with linear weight generating functions and another with quadratic weight generating functions. For example, for the following linear function,

$$l(x) = \alpha \frac{\beta x + 1}{1 + \beta}, \quad \text{with} \quad x \in [0, 1] \quad \text{and} \quad 0 \leqslant \alpha, \beta \leqslant 1$$

an illustrative example for semantic usage (e.g. to elicit the relative importance of criteria in a semantic format) is shown in Table 1. The assigned values to each parameter are indicative (recommended) but they can be changed/tuned according to the application.

The α parameter provides the top limit for each semantic importance and the weighting functions – decreasing in this case – will establish the lower limit using the β parameter. Since β provides the slope for the weighting functions, and this depends on the criterion at hand, the reasoning is that the higher the β the steeper the function is. Fig. 1 depicts graphically three examples: very important, important and low importance, using, respectively, for slopes parameters: medium, low and high.

It should be pointed that an extended formulation for the mixture operators with weighting functions is proposed in the FIF algorithm to allow dealing with lack of confidence and inaccuracies in the input data – quite a common occurrence in hazard maps (i.e. criteria). Details about this extension will be discussed in step 4 of the FIF algorithm (Section 3.6).

3. Fuzzy Information Fusion (FIF) algorithm

In this section we present the proposed fuzzy information fusion algorithm. We begin by the context and scope of the developed work and then we explain its steps using two hazard maps to illustrate the approach.

3.1. Context and scope[1]

There are a number of issues that make information fusion a challenging task. We start with the important question posed by [32]: "How to obtain a final value for each potential solution (e.g. pixel, hypothesis) from combining different heterogeneous measurements m_j^i? " The authors propose using three stages:

[1] In this section we will mention, interchangeably, alternative, pixel or candidate solution (e.g. maps/matrices with 10 × 10 pixels include 100 candidate solutions).

(i) Transform the measures in such a way that it is possible to combine them.
(ii) Combine the data as transformed by the representation according to the allowed rules for the chosen framework (e.g. Bayes rule).
(iii) From the resulting combination take a decision in agreement with the problem.

From our point of view, information fusion is only related with the first two steps: representing, transforming, cleaning and aggregating information. The last one (iii) is already in the realm of ranking or selecting candidate alternatives (decision) and it is outside the goal of obtaining a single composite of fused information.

In addition, information fusion should comply with important critical success factors [30], such as: (1) robustness; (2) extended spatial–temporal coverage; (3) high confidence, (4) low ambiguity; (5) reliability and validity; (6) low vulnerability. Other authors [28] also pointed different critical factors, such as data correlation, data alignment, static versus dynamic phenomena and so forth. However, these challenges are mostly related with sensor fusion and therefore not applicable in the FIF context. Summarizing, the FIF (Fuzzy Information Fusion) algorithm incorporates the two stages (i) and (ii) and the critical factors (1)–(6). We believe that FIF is a general information fusion algorithm suitable for tackling uncertain environments, particularly when there is a lack of confidence and inaccuracies on the input data, as well as, when the criteria are quite heterogeneous and require intelligent normalization.

As mentioned in the introduction, the preliminary research for devising this algorithm was done in the context of past research projects, which main goal was to recommend an adequate interplanetary spacecraft target-landing site. The projects main publications are the following [5,36,53]. In these publications hazard maps were modelled as fuzzy functions to become the inputs for a dynamic fuzzy multi-criteria model with feedback and an optimization process to answer real-time requirements. The preliminary works implied dynamic environments, several iterations and feedback information, while here the scope is restricted to fusing information at a certain time *t*, without any concerns about past information. Two hazard maps (slope and texture) from the aforementioned projects are used to illustrate the FIF algorithm.

3.2. FIF algorithm architecture

In Fig. 2 it is depicted the complete architecture of the proposed FIF algorithm. As explained before, two hazard maps (“low slope” and “low-variance texture”) are used for illustrating the steps of the FIF algorithm. However, any other data sources that could be normalized with membership functions could be used.

Observing Fig. 2 we can see that the step filtering uncertainty is shown as independent of the assignment of relative importance of criteria. However, in FIF we strongly support that one of the best aggregation operators for fusing information is a mixture operator with weighting functions, because it allows rewarding or penalizing badly satisfied criteria. By extending this operator to include the uncertainty filtering the three steps after normalizing are jointly taken in consideration. We chose to depict the figure in this way to convey that any other suitable aggregation operator could have been chosen and that the four proposed steps for FIF are essential to achieve a successful fusion of information. Hence, the proposed FIF algorithm includes 4 main steps:

(1) Normalization process, which includes a mathematical transformation (fuzzification) of maps to ensure numerical and comparable data for fusion.
(2) Filtering uncertainty from data regarding inaccuracies and lack of confidence in input data (e.g. hazard maps matrices have embedded imprecisions).
(3) Assigning relative importance to each criteria membership value, which depends on the satisfaction/suitability of criteria for a specific alternative.
(4) An aggregation/fusion method (i.e. aggregation operator) for combining all matrices (criteria) into a single composite (e.g. fused maps for an iteration).

In the next sub-sections we describe in detail its steps.

3.3. Step 1. Normalization with membership functions (Fuzzification)

Considering the heterogeneous matrices of hazard maps (i.e. set of criteria to be fused) they first need to be normalized in order to be numerically comparable and manipulated by the fusion process. Our proposal is to normalize data using fuzzy membership functions, i.e. fuzzification process [44]. Besides guaranteeing normalized and comparable data, a fuzzification method allows

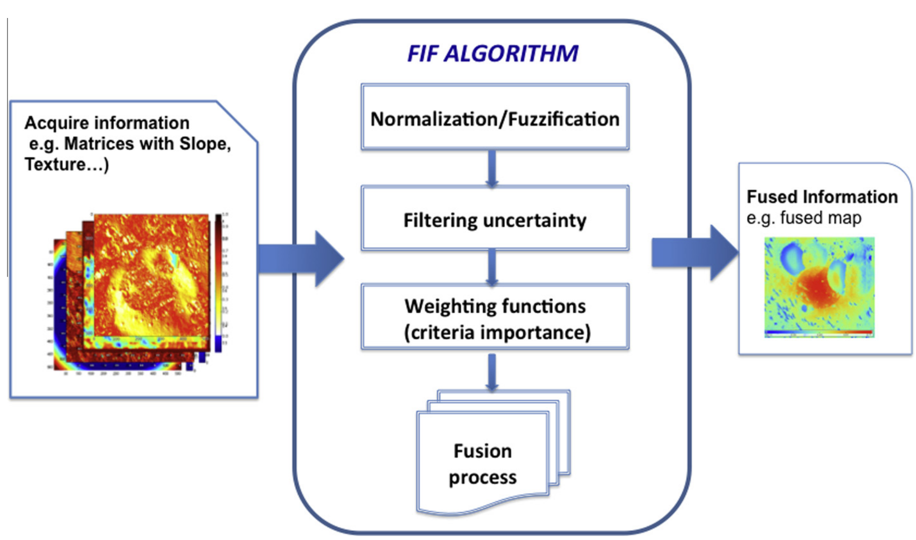

Fig. 2. FIF algorithm architecture.

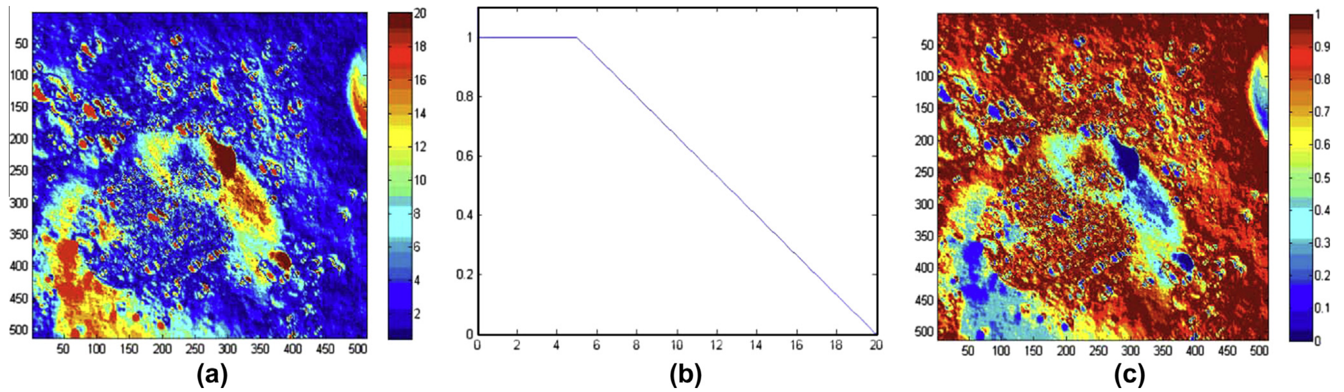

Fig. 3. Example of normalization (Fuzzification) of criteria low-slope. (a) Original hazard map; (b) membership function topology; (c) normalized map.

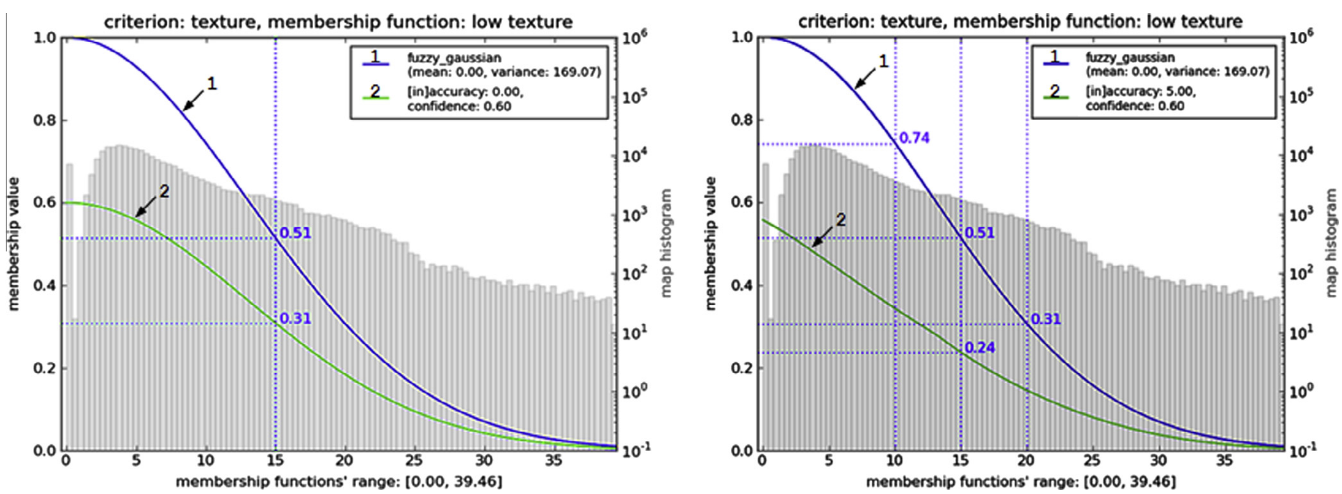

Fig. 4. Example of filtering "low texture" with lack of confidence and inaccuracy.

representing data with semantic concepts, such as "low slope" (e.g. lower than 20°) or "low-variance texture".

An important problem on the variables "fuzzification" is to choose the best topology for the membership functions, because we have to take into account the context objective. In the case of hazard maps for safe landing the objective is to choose the "safest" and scientifically more promising site for landing (e.g. low slope, good-illumination, low-variance texture). Hence, topologies such as triangular, trapezoidal or Gaussian membership functions should be chosen depending on the nature of the criterion and the desirable interpretation (semantic) for each criterion.

An illustrative example of a specific hazard map iteration is shown in Fig. 3. The membership function represents "low slope", where slopes should be lower than 20°. This criterion is normalized ("fuzzified") with an open triangular membership function defined as follows:

$$\mu(x) = \begin{cases} 1 & \text{if} \quad x \le c \\ \frac{20-x}{20-c} & \text{if} \quad c < x \le 20 \end{cases}$$

where $x \in [0,20]$; $c = \min(x) + \alpha \cdot (\max(x) - \min(x)) = 0.05 * 20 = 1$, α is the range of the function plateau.

In this case we consider $\alpha = 0.05$ (which results in a function plateau in the range [0,1]) but the parameter can be adjusted. The min and max values are extracted from the input matrix.

Observing Fig. 3 we can see that, after normalization, the hazard map representation (c) displays "good" places painted in [2]red (i.e. with low slope), and dangerous places (high slope) in blue. The color scales are shown in the right side of each map both for the input and output. With this process, for each x (criteria value in the input information) we obtain a membership value, i.e. its fuzzified score, representing how well the alternative (x) satisfies the criteria.

In addition, please notice the bar-charts in Fig. 4, which represent the histogram of all values in the input matrix of "low texture". Its topology was used to define the respective membership function. These examples show two different modes of fuzzifying the input matrices, one with expert knowledge (e.g. "low slope", Fig. 3) and another through a histogram to obtain the suitable typology (e.g. "low-variance texture", Fig. 4). However, there are many other methods to define membership functions (see for example [44]).

3.4. Step 2. Filtering uncertainty

In this section we address the intrinsic uncertainty that can be found in the input information to be fused. For any given alternative, each criterion must be adjusted by decreasing its membership value accordingly to the lack of confidence and inaccuracy in the

[2] For interpretation of color in Figs. 3, 5 and 6, the reader is referred to the web version of this article.

input data. This step is performed with a twofold filtering function: (a) combines metrics to deal with both types of uncertainty in the input values; (b) reflects the attitude of the decision maker [36].

Lack of confidence affects all input values regarding their membership values and inaccuracy creates an interval with left and right deviations from the initial value. We selected this function because it enables to adjust (decrease) the membership functions to reflect the embedded input information uncertainty and also to incorporate a pessimist or optimist view of a developer. Formally, the accuracy and confidence parameters (a_{ij} and wc_j respectively) will modify the membership functions' values using the following expression for the filtered uncertainty:

$$fu_{ij} = wc_j * \left(1 - \lambda * \max_{x \in [a;b]}\{|\mu(x) - \mu(x_{ij})|\}\right) * \mu(x_{ij}) \qquad (1)$$

where x_{ij} is the value of the jth criterion for site i; $\mu()$ is a membership degree in a fuzzy set; wc_j is the confidence (percentage) associated to criterion j; and $[a, b]$ is the inaccuracy interval, defined as follows:

$$a = \begin{cases} \min(D) & \text{if} \quad x_{ij} - a_{ij} \leq \min(D) \\ x_{ij} - a_{ij} & \text{if} \quad x_{ij} - a_{ij} > \min(D) \end{cases}$$

$$b = \begin{cases} x_{ij} + a_{ij} & \text{if} \quad x_{ij} + a_{ij} \leq \max(D) \\ \max(D) & \text{if} \quad x_{ij} + a_{ij} > \max(D) \end{cases}$$

where a_{ij} is the accuracy associated to criterion j for site i, and D is the variable domain. Further, $\lambda\,[0,1]$ is a parameter to reflect the optimistic or pessimistic attitude of a decision maker, where close to 1 indicates a pessimistic attitude and close to zero an optimistic attitude.

Despite its name, the accuracy value a_{ij} represents a deviation from a central value, indicating the amount of inaccuracy in the observations. It indicates that any value x_{ij} is included in the interval $[x_{ij} - a_{ij}, x_{ij} + a_{ij}]$. In the fusion algorithm we consider two types of inaccuracy values: absolute and relative. Absolute values, the most common, are those where the inaccuracy is independent of the input value: $a_{ij} = a_j$. Relative values are those where the accuracy value is a function of the input value x_{ij}. These will take the form of $a_{ij} = a_j * x_{ij}$.

An example of how the filtering may work can be seen in the membership function for "low texture", displayed in Fig. 4. In the left function we see the decrease caused in the membership function by having a confidence on the input data of only 60% but without any inaccuracy. In the right function we see the further decrease in the membership function by considering an inaccuracy (deviation interval) of 5 (note: in this case the inaccuracy interval is [0, 10]. In both examples we used a pessimistic attitude from the decision maker, i.e. $\alpha = 1$.

As it can be observed, we have in blue the membership function representing the fuzzy set "Low texture" and in green the adjusted function after using the *Filter*. In this example, to a "low-texture" value of $x = 15$ corresponds a membership value of $\mu(15) = 0.51$. When we filter the information with a confidence of 60% the membership value decreases to $\mu(15) = 0.31$ (left graphic). If we further filter the information with an (in) accuracy deviation of 5 (the range in the example is [0, 10]) the corresponding value decreases to $\mu(15) = 0.24$ (right graphic). Note that the bar-charts in both graphics represent the histogram of all values in the input matrix that were used to define the membership functions topology.

In this fashion we can penalize all input values affected by any kind of uncertainty (inaccuracy or confidence). Obviously these parameters can be customized for any information fusion problem.

3.5. Step 3. Relative Importance of criteria with weighting functions

In FIF we propose the use of linear weighting functions [37,43] to express the relative importance of criteria. The rationale of these weighting functions is that the satisfaction value of a criterion should influence its pre-defined relative importance. This weighting system proved quite satisfactory for dealing with selecting the best place for landing [36] because they could be changed along the descent and synergistically penalized badly satisfied sites.

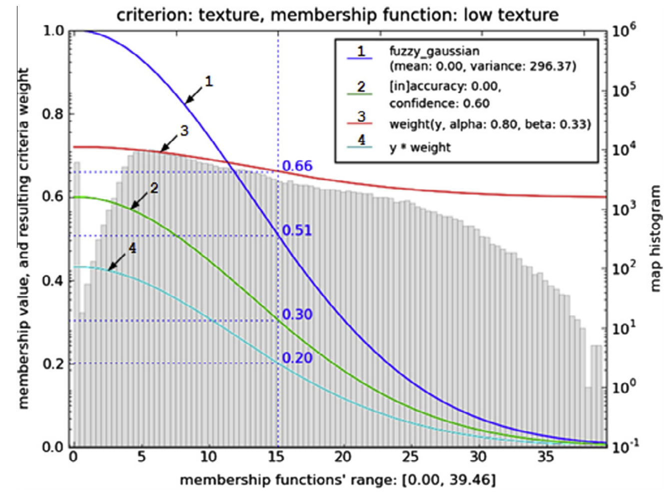

Fig. 5. Example of hazard map importance assignment with weighting functions.

For the FIF algorithm we propose to use a modified linear function $L(x)$, to increase the computational efficiency and understandability, as follows:

$$L(fu_{ij}) = \alpha \frac{1 + \beta fu_{ij}}{1 + \beta}, \quad \text{where} \quad 0 \leqslant \alpha \leqslant 1 \quad \text{and} \quad 0 \leqslant \beta \leqslant 1 \qquad (2)$$

where α_j, $\beta_j \in [0,1]$ and fu_{ij} is the *accuracy & confidence* membership value from Eq. (1) for the jth criterion of alternative i.

We can see that the parameter α is used to weight the relative importance of the different criteria. The parameter β controls the ratio $L(1)/L(0) = 1 + \beta$ between the maximum and minimum values of the effective generating function. When we have $\beta = 0$ (the weighting function does not depend on criteria satisfaction) it falls in the classical weighted average aggregation operator.

The definition of the weighting functions morphologies is given by the parameters α and β, according to what was explained in Section 2.2.1). As exemplified in Table 1, the α parameter provides the semantics for the weighting functions as for example (very-important, important, low importance, etc.). The β parameter provides the required slope for the weighting functions to enable more or less penalization or rewarding. The α and β parameters will be set depending on the initial assigned semantic importance (e.g. important = 0.8) and on the sharpness of the function decrease, which depends on how much we want to penalize the badly satisfied criteria.

Using the same illustrative example of "Low variance texture" (Fig. 4), we exemplify in Fig. 5 the process of using weighting functions to express the relative importance of this criterion. The red line represents the defined weighting function with parameters $\alpha = 0.8$ (important criteria) and $\beta = 0.33$ (low decrease for the weight). It is easy to observe that for a texture of 15 units (x-axis) the initial membership value is 0.51. After filtering, the uncertainty in the previous step (in Fig. 5 we only consider lack of confidence of 60% and 0 for inaccuracy) we go down to 0.30. After weighting the relative importance of the criteria, the final value to be used for the fusion is 0.20; i.e. we highly penalize the variable satisfaction value (0.51) because it displayed relatively low performance and our confidence in the data was relatively low.

The motivation for using weighting functions in the FIF algorithm is to enable rewarding or penalizing criteria (e.g. slope, texture) and also to remove (filter) the imprecision in the input data regarding lack of confidence and possible inaccuracies.

3.6. Step 4. Fusion process (aggregation)

In this subsection we focus on the fusion of the transformed input information from various sources (in this paper illustrated with hazard maps). The information fusion aggregation method proposed for fusing information is based on the mixture of operators with weighting functions [37,43], and its general formulation is:

$$r_i = \oplus(W(fu_{i1}) \otimes ac_{i1}, \ldots, W(fu_{in}) \otimes fu_{in}) \quad (3)$$

where $\oplus$ is an aggregation method (e.g. sum, max, parametric operators); $\otimes$ is a conjunction operator (e.g. multiplication, min); fu_{ij} is the filtered uncertainty *accuracy & confidence* membership value of the jth hazard-map for solution i (Eq. (1)); $W(fu_{ij}) = \frac{L(fu_{ij})}{\sum_{k=1}^{n} L(fu_{ik})}$ where $L(fu_{ij})$ is the weighting function above (Eq. (2)).

As can be observed, the weighting function of this mixture operator was extended to include dealing with imprecision (explanation in Section 3.5). The result of this step concludes the information fusion steps of the FIF algorithm.

An illustrative example of FIF with 2 hazard maps, "low slope" and "low-variance texture", is depicted in Fig. 6.

The two maps to be fused, displayed in Fig. 6, are raw input maps and the respective color scale in the right shows "good low-slopes" and "good low-variance-textures", with red for low values (objective for the fusion) and blue for good values. The fused hazard map (right) uses the same logical color scheme, i.e. "good" alternatives are redder and bad ones are in bluer. The latter scale corresponds to the membership values within the interval [0,1]. It should also be noticed that the fused hazard map (Fig. 6right) represents the input for the decisions-making process, i.e. it is the complete search space including all the potential alternatives for safe landing sites. In this map redder pixels correspond to best landing spots; bluer pixels represent worst landing alternatives.

3.7. Comparative study

Computational intelligence (CI) focuses on adaptive techniques to enable or facilitate intelligent behavior in complex and changing environments [13]. Basically, CI is geared towards dealing with imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and computational efficient solutions. Key areas in CI are artificial neural networks, evolutionary computing, swarm intelligence and fuzzy systems [13]. Evidential reasoning, possibility theory and rough sets are other well-known tools for dealing with imprecision on data fusion systems [28,30] and they are plausible substitutes for fuzzy sets in CI. A good overview on this subject can be seen in [28].

On one hand, there are several applications of CI approaches for data fusion, but not applied to hazard avoidance (see for example [11]). On the other hand, there are classical approaches for Hazard Detection and Avoidance (HDA), which do not use CI approaches (see for example [27,61]). Due to the wide range of proposed methods for information fusion here we will limit our comparison to CI approaches for information fusion applied to hazard detection and avoidance. Moreover, the comparison is mostly theoretical because there are no benchmarks and the FIF algorithm. Table 2 presents a comparison of the main characteristics of the computational intelligence approaches for hazard mapping fusion.

As can be observed in the table above, there are three main types of CI mechanisms, fuzzy sets, neural networks and evolutionary computation with good characteristics for handling information fusion for safe landing with hazard avoidance. We classified the fuzzy approaches with highly applicable, mostly because it is quite a versatile mechanism to handle most data uncertainties and can be embedded in different paradigms, from rule-based systems to multi-criteria decision systems, as shown in the references. Moreover, fuzzy approaches do not need any training, as required by all neural networks approaches, which for an (mostly) unknown planetary landing is a huge asset. Regarding evolutionary approaches the main problem is the lack of guarantee about the "optimal" solution. However, there are no "free lunches" and there is always a trade-off between computational efficiency and optimality.

From the point of view of the FIF algorithm, a comparison is:

(a) FIF is more adaptable than the fuzzy rule-based approaches because it does not require re-doing the knowledge-base and it is mostly context-independent (only the normalization process – step 1 – it may need some expert knowledge to assess what is good or bad).
(b) FIF does not need any training sets (as ANN approaches need) to tune the algorithm. Further, it is adaptable to any environment (Planets, Asteroids, Earth...) without any training. Further, FIF can handle uncertainty in the input data while ANN approaches usually are not robust on changing environments.
(c) FIF is able to handle and represent the imprecision and uncertainty involved in the input data, while evolutionary approaches need fixed assumptions. Further, evolutionary approaches are not prepared for multiple objectives (criteria) and require some sort of previous aggregation in these cases (the FIF is a multiple criteria system per se).

From this comparison we may say that hybrid approaches will certainly contribute to improvements in data fusion because each type has its own advantages and together they can enable synergies to achieve better results. FIF demonstrates this contribution. Finally, to the best of our knowledge there are few real-time applications for hazard detection and avoidance using CI algorithms and

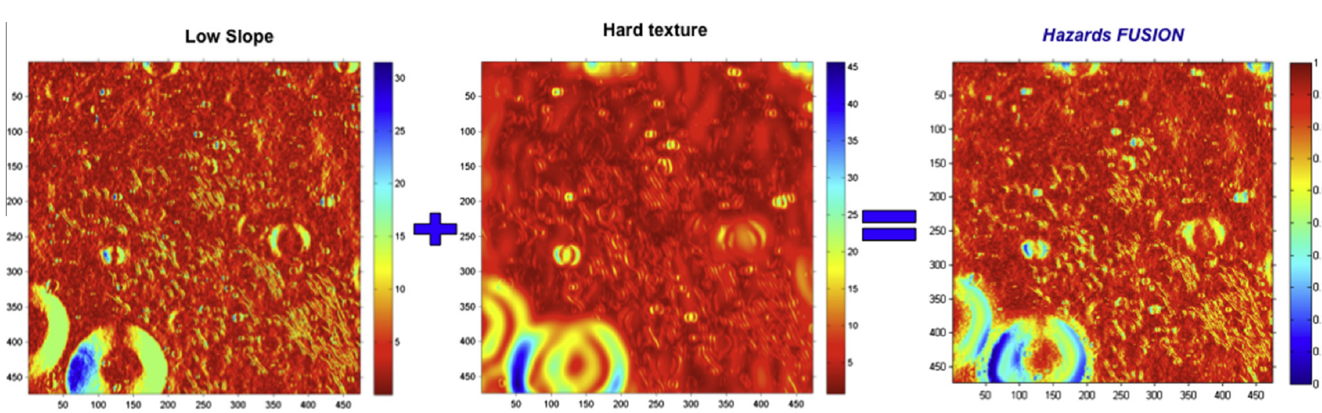

Fig. 6. Example of fusion of low-slope and low-variance texture hazard maps.

more validations and comparisons will be needed to properly assess these technologies, however, we believe hybrid approaches are the best strategy and our preliminary work [53] demonstrates the versatility of this hybrid approaches.

4. Final comments

Many approaches for high level fusion (i.e. information fusion), proposed in the literature, do not distinguish the fusion process from the decision making process, where the latter implies selecting the "optimal" alternative. From our point of view, the objective of fusion of information is to obtain a combined score/value for each alternative and does not include the decision task of selecting the "optimal" alternative. This distinction – provided by the multi-criteria decision making paradigm – is important because we can use different optimization methods to obtain a ranking such as simple ordering, dominance methods, cross comparisons, and evolutionary optimization algorithms. For example, in the case study of selecting a safe place for landing spacecraft [53] – from where the two illustrative hazard maps used in this work were borrowed – it was used a hybrid evolutionary algorithm, resulting from combining particle swarm optimization (PSO) and Tabu search algorithms to select the "optimal" landing place. This non-exhaustive selection process resulted in huge computational gains, decreasing from 1.31 s to around 4 ms per iteration.

Although there are many efforts to develop fusion approaches (see good surveys in [30,28]) there are still important issues to be addressed. Particularly, when we talk about information fusion in uncertain environments, where input information may include quantitative criteria (e.g. hazard maps) and qualitative criteria (e.g. scientific interest) there are not many answers. The proposed Fuzzy Information Fusion (FIF) algorithm is a step forward in this direction since it is able to cope with both types of criteria.

Further, FIF fully addresses the three main challenges of any information fusion process: data must be numerically comparable; imprecision and uncertainty must be taken in consideration; a suitable aggregation operator must be selected to combine the information. Moreover, FIF is general enough to be applied to any problem, where the inputs may be from many different sources, as long as they can be modelled as fuzzy sets representing a semantic concept (e.g. "low-slope"). FIF's versatility also allows customization and tuning of the chosen parameters for expressing relative importance, as well as for the aggregation function (fusion).

In summary, this paper introduced the FIF (Fuzzy Information Fusion) algorithm, which combines concepts from both computational intelligence and multi-criteria decision making areas. We discussed the theoretical aspects of each of the four proposed steps: normalization/fuzzification; filtering uncertainty; weighting (importance) criteria; and fusion (aggregation). Along the step's description we used an illustrative example (two hazard maps) to clarify the algorithm. Finally, we showed how FIF could be used as a versatile mechanism to handle heterogeneous data, normalize it, and produce a fused information aggregation, ready for supporting effective decision making.

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